Bidirectional Multi-Hop Communication via Two Relays Using Nested Voronoi Codes

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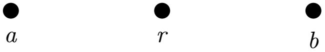
Outline

1. Basic Principles
2. One Relay (Separated Two-Way Relay Channel)
3. Two Relays (Separated Two-Way Two-Relay Channel)
4. Conclusion

Message Exchange via a Relay

Message Exchange via a Relay



- Three nodes / devices

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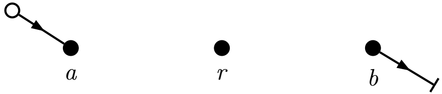

- Three nodes / devices
- One message from a to b

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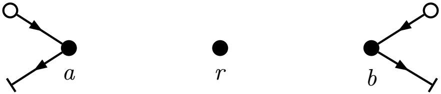

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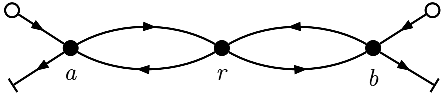

- Three nodes / devices
- One message from a to b
- One message from b to a
- No direct link

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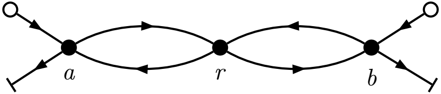

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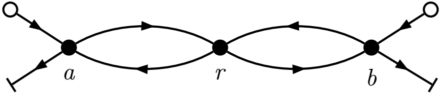

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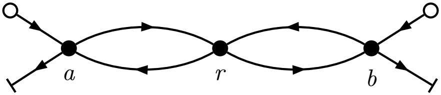

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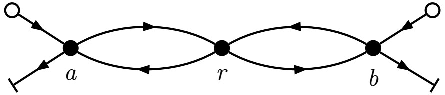

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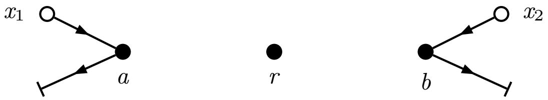
Start with toy problems:

- No noise at the nodes
- Binary input and output alphabets
- Half-duplex constraint (i.e. a node either listens or talks but not both at the same time)

Routing

Exchanging 2 bits x_1 and x_2 with a routing approach:



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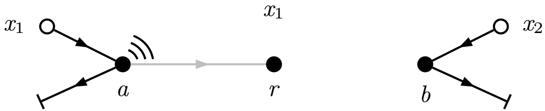

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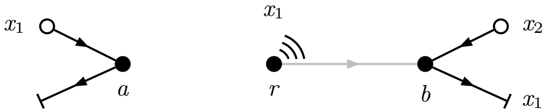

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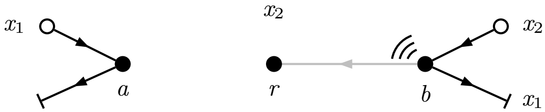

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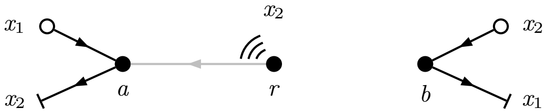

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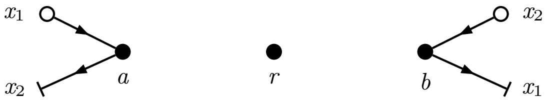

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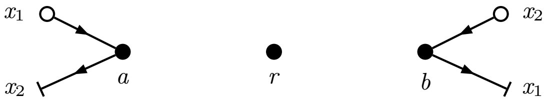
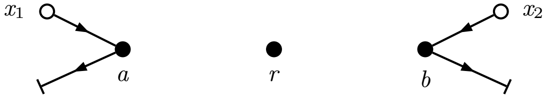


sum rate: 2 bits in 4 transmissions

Network Coding

The relay node r can reach both a and b. Broadcasting saves one transmission.



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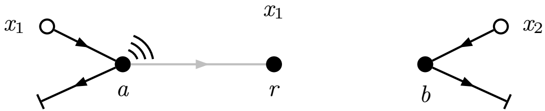

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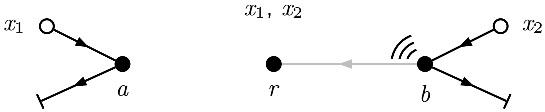

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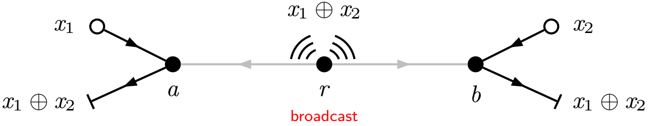

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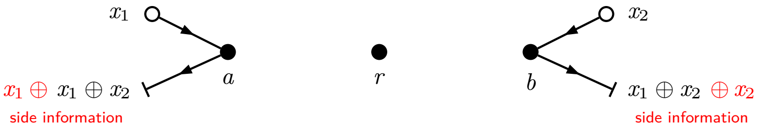

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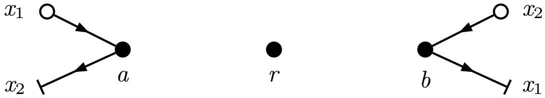

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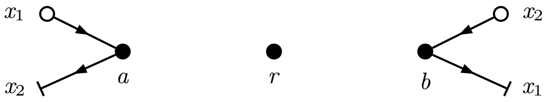
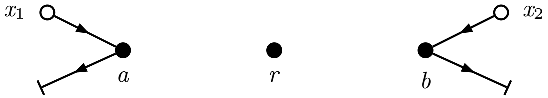


sum rate: 2 bits in 3 transmissions

Physical-Layer Network Coding

The relay node doesn't need to know the individual bits.



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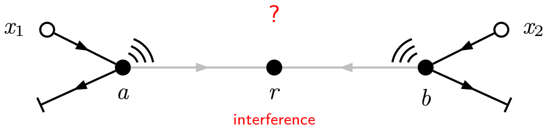

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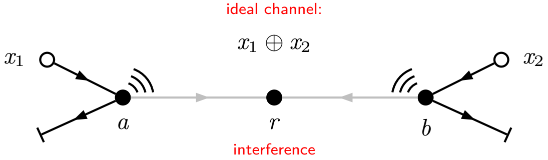

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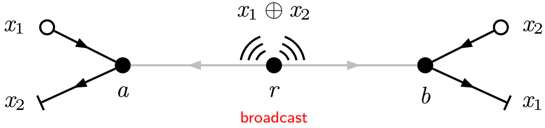

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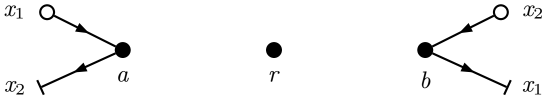

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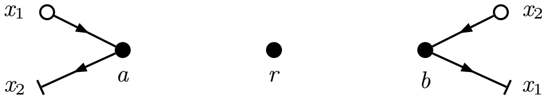
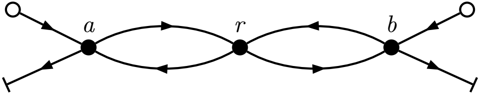


sum rate: 2 bits in 2 transmissions (for ideal channel)

Obvious Question

How does it work for more realistic channel models?

Gaussian Channel Model



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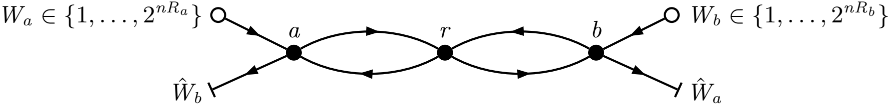

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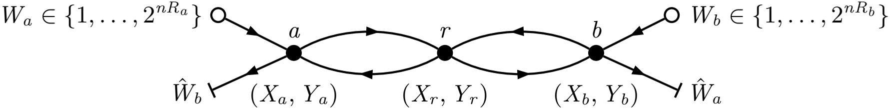

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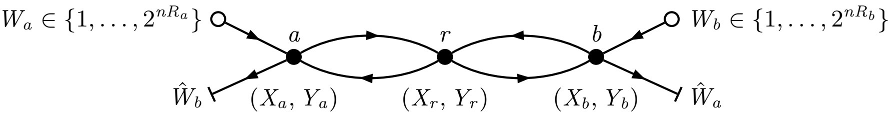

Additive Gaussian noise:

$$Y_a = X_r + Z_a$$
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Successive channel uses:
$\boldsymbol{X}_a = (X_{a,1}, \ldots, X_{a,n})$, etc.

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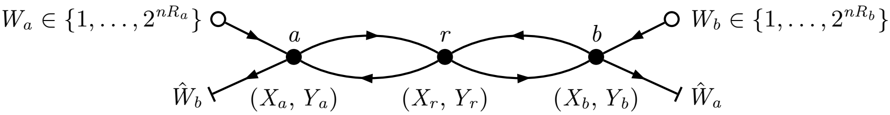

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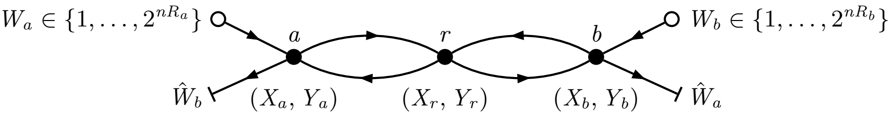

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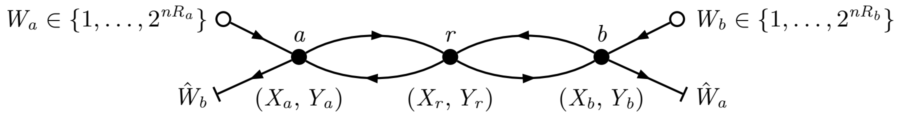

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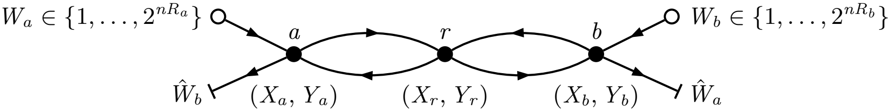

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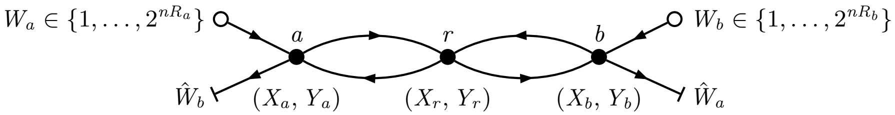

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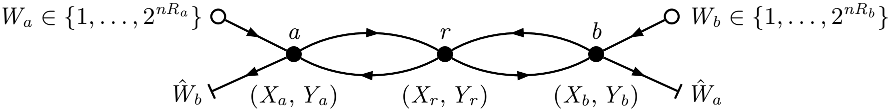

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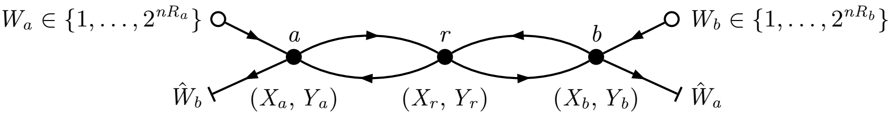

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What (R_a, R_b) are achievable ($p_e \to 0$ as $n \to \infty$)?

Decode-and-forward (DF)

See [Rankov-Wittneben '05], [Knopp '06].

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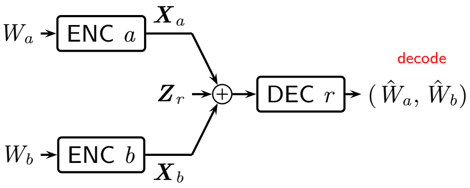

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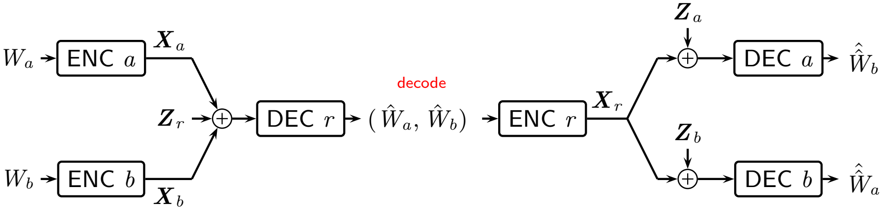

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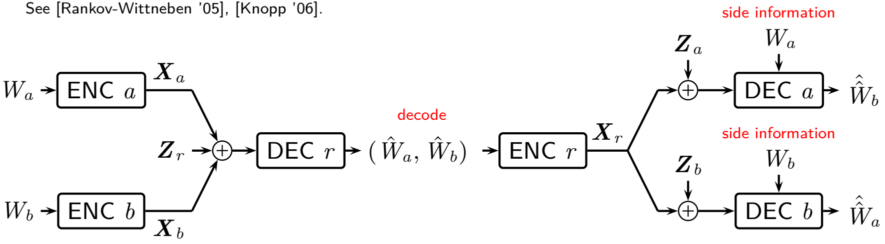

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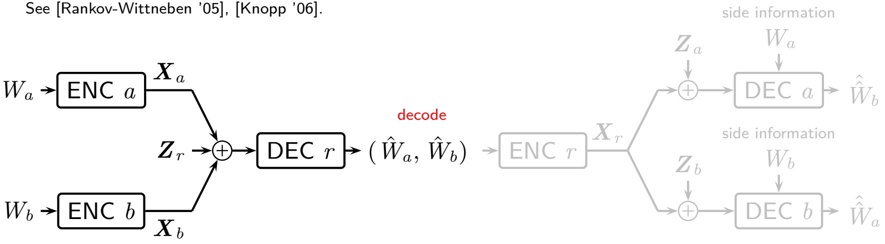

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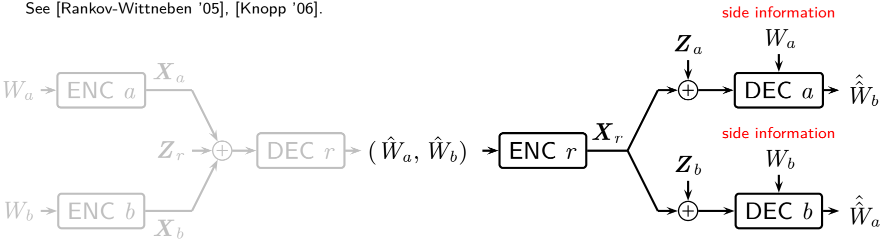
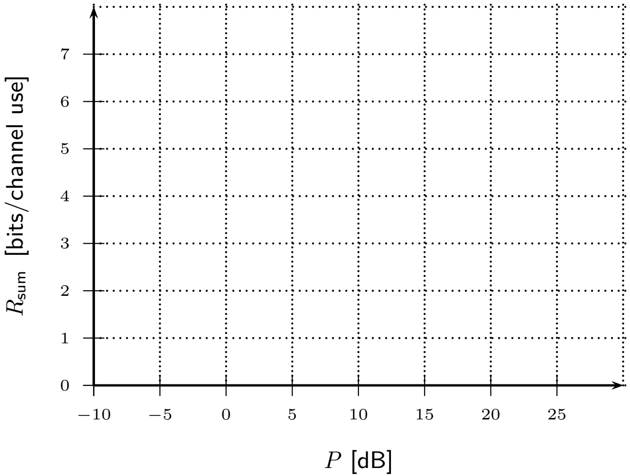


- Multi-acces channel
- Broadcasting with user side information

Sum Rate $R_{\text{SUM}} = R_a + R_b$



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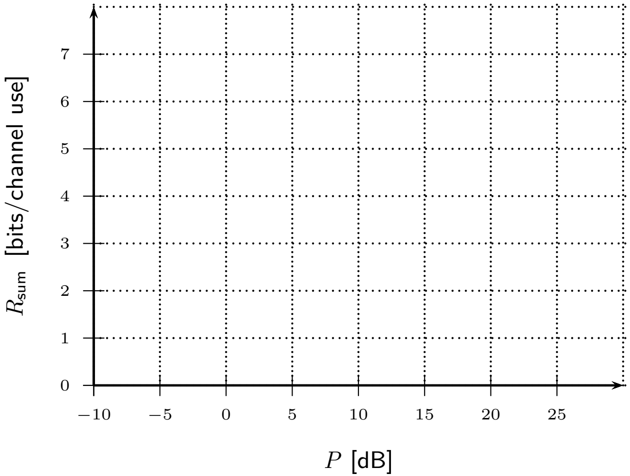
Symmetric case: $P_a = P_r = P_b = P$ and $\sigma_a^2 = \sigma_r^2 = \sigma_b^2 = 1$



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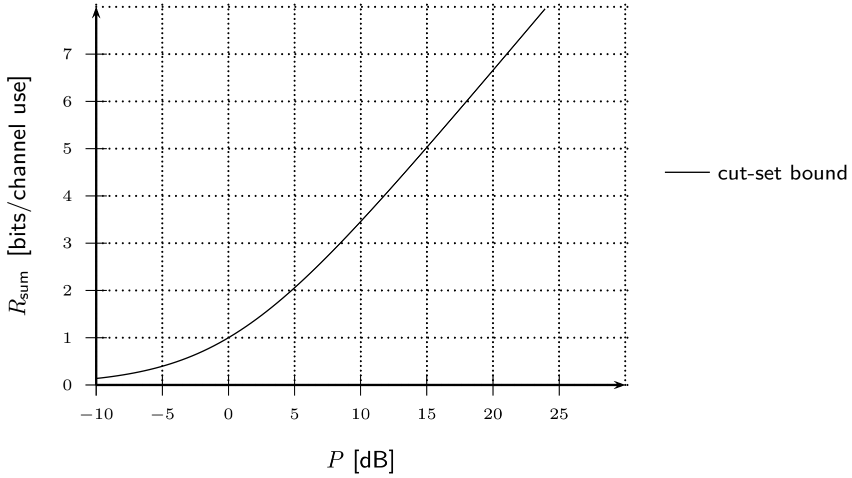

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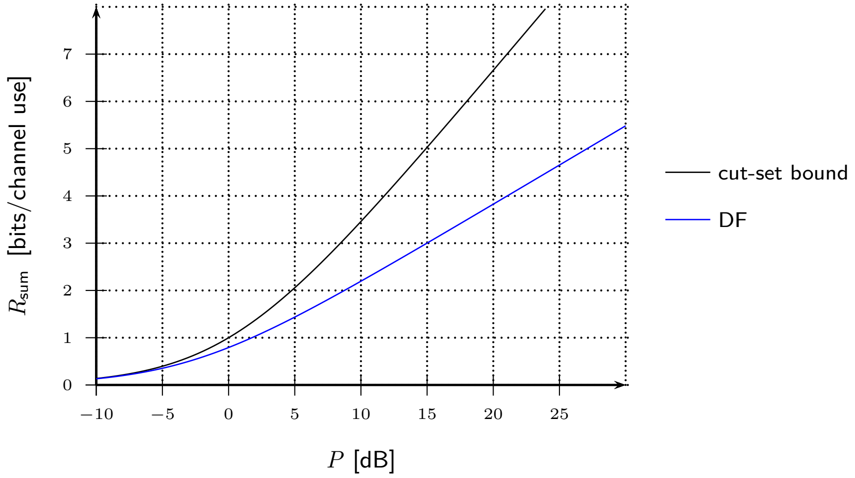

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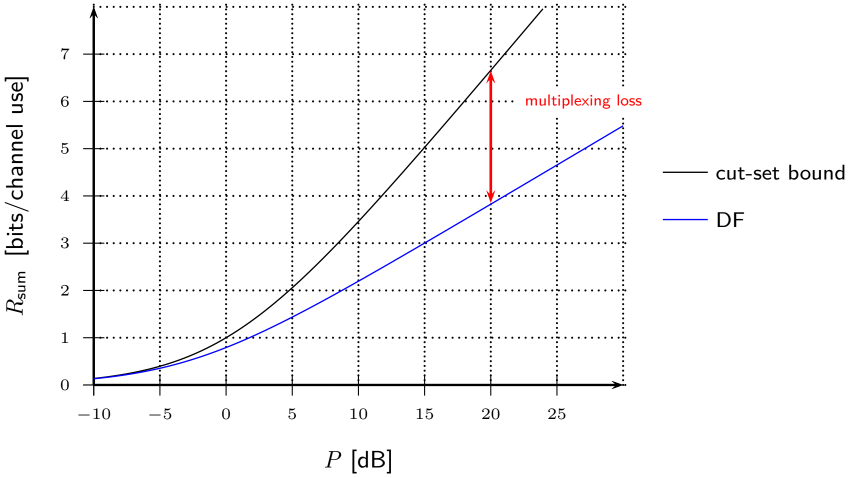

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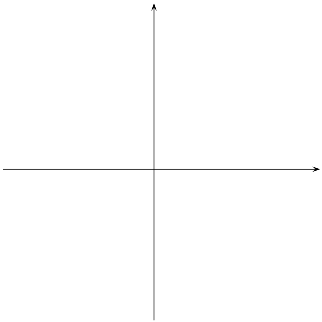
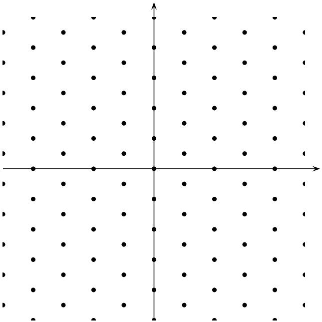
- PNC idea: the relay should only try to understand a combination of the transmitted messages.
- Use structured codes: Voronoi codes

Lattices and Voronoi Codes



Lattices and Voronoi Codes

- (coding) Lattice Λ_c



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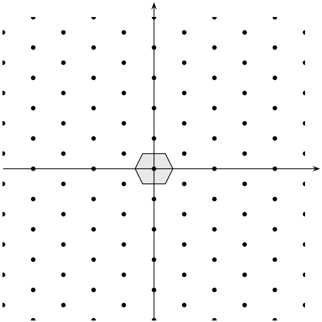

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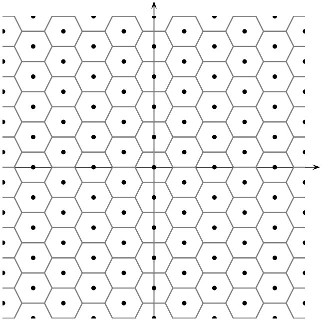

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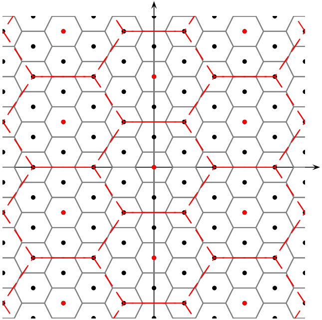

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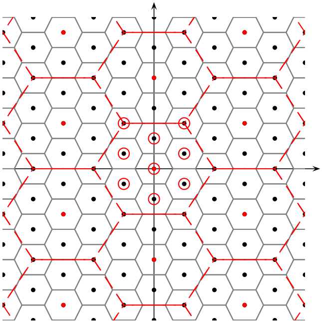
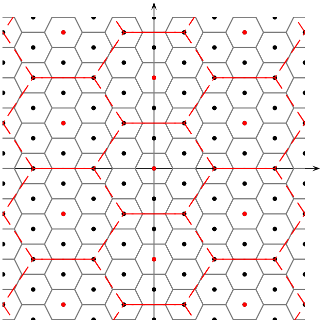


The Basic Idea



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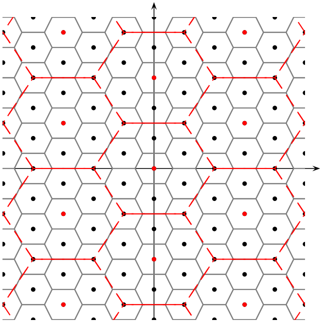
- Users a and b use the same Voronoi code $\mathcal{C}(\Lambda_c/\Lambda)$.



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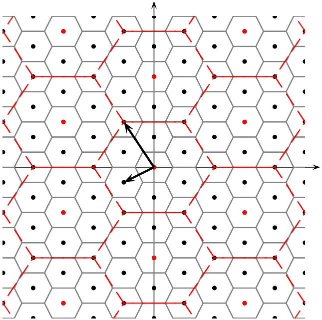

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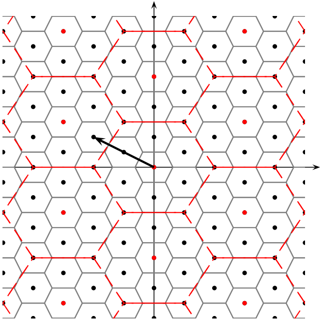

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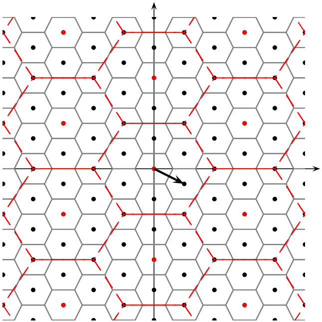

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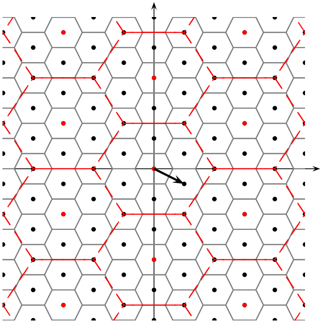
- Side information allows each user to recover the lattice point of the other user based on $\boldsymbol{V}$.



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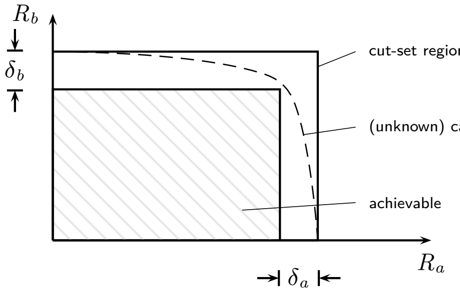

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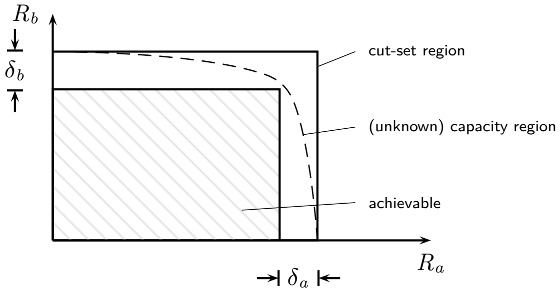

Achievability strategy:

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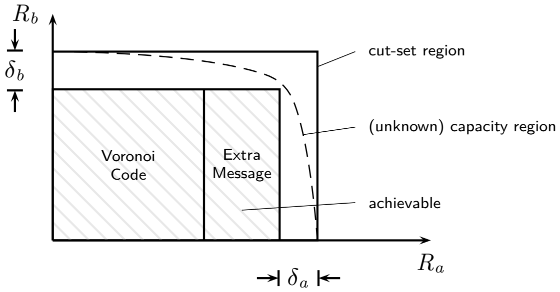

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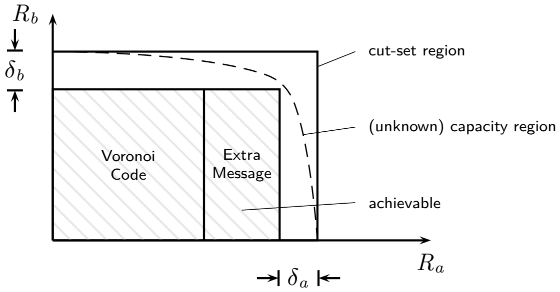

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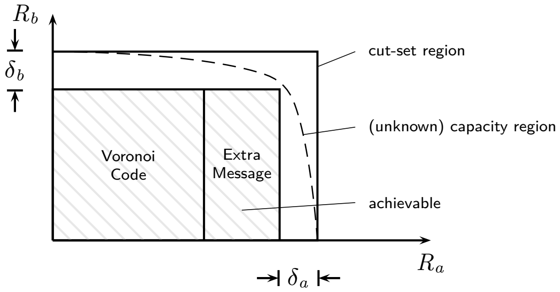

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(see also [Nam-Chung-Lee '10])

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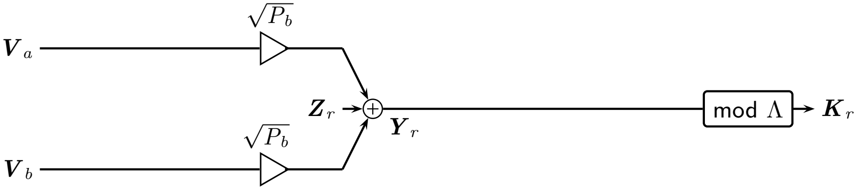
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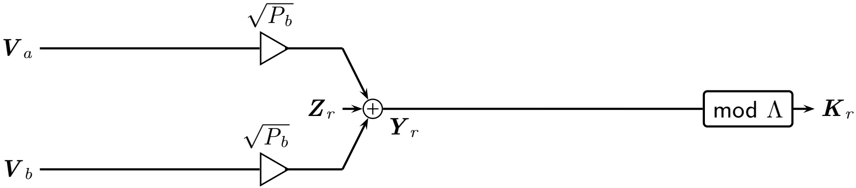
$\boldsymbol{V}_a$
$\boldsymbol{Z}_r$
$\boldsymbol{Y}_r$
mod Λ
$\boldsymbol{K}_r$
$\sqrt{P_b}$
$\boldsymbol{V}_b$

Uplink



- Lattice decoding: $Q_{\Lambda_c}(\boldsymbol{K}_r)$

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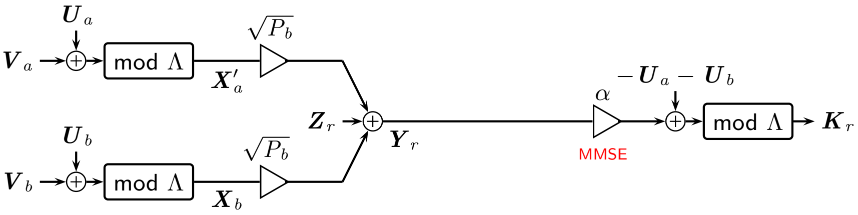

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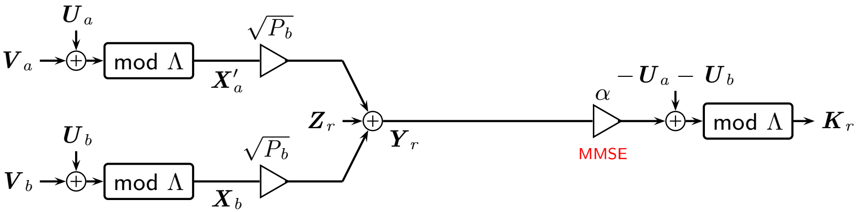

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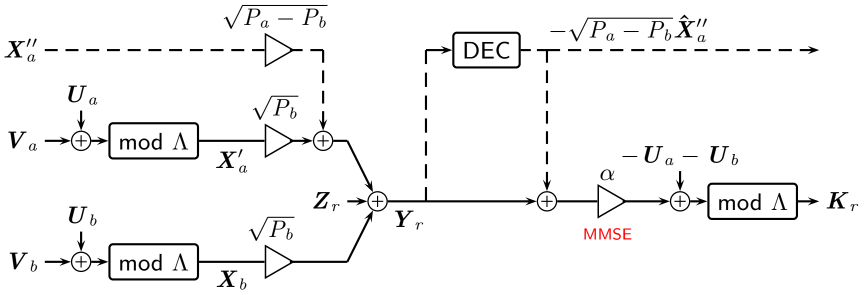
Uplink



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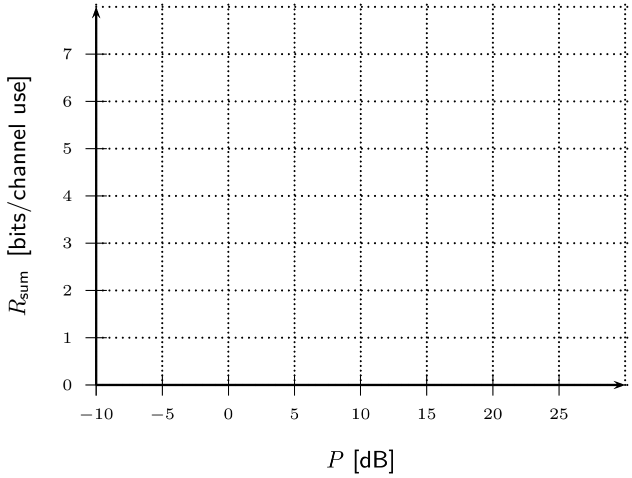
SUM RATE $R_{\text{SUM}} = R_a + R_b$



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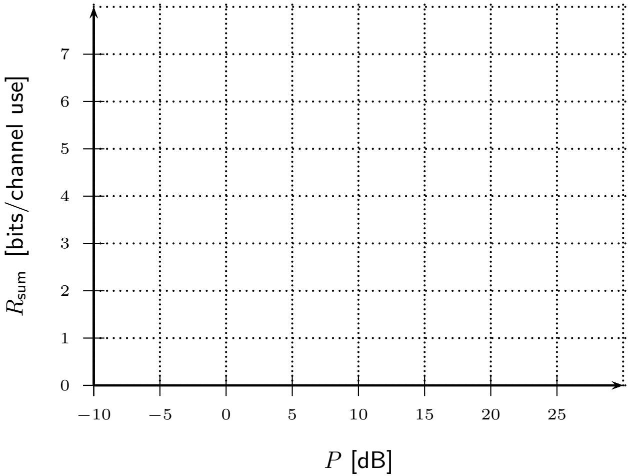
Symmetric case: $P_a = P_r = P_b = P$ and $\sigma_a^2 = \sigma_r^2 = \sigma_b^2 = 1$



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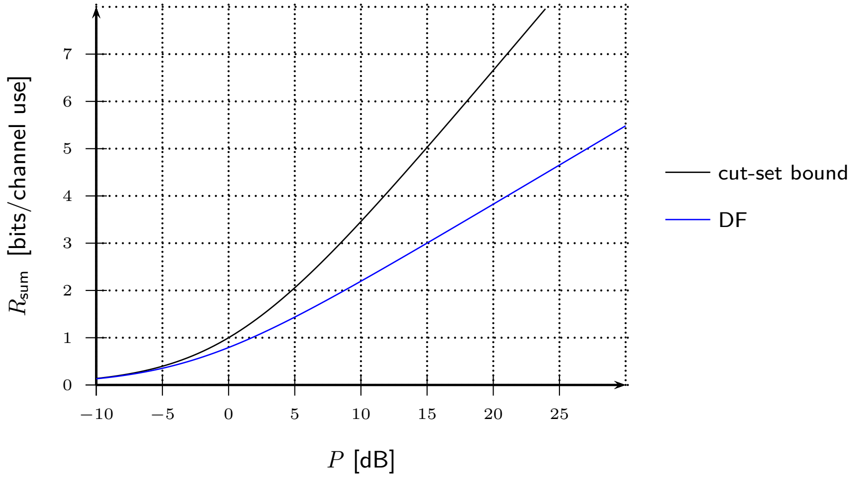

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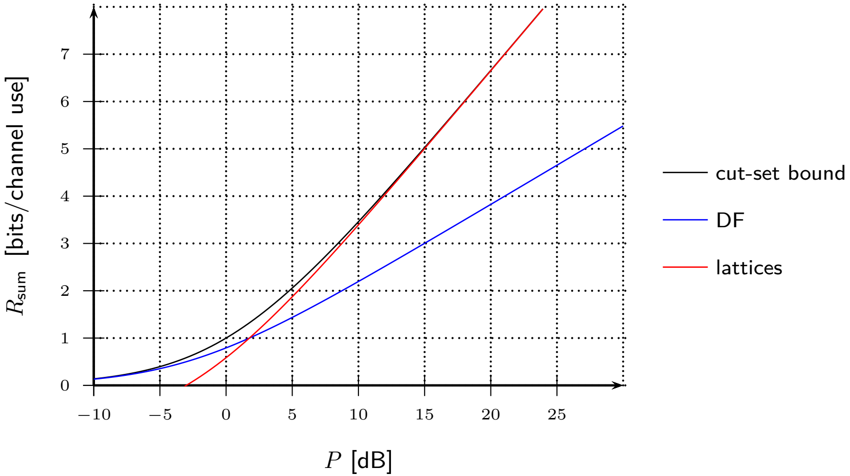
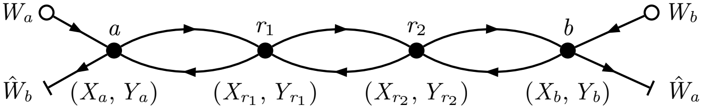


The Separated Two-Way Two-Relay Channel



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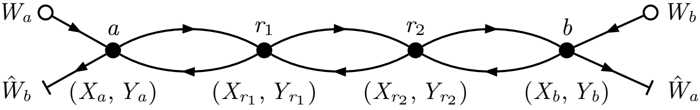

- Multi-hop extension of the separated two-way relay channel

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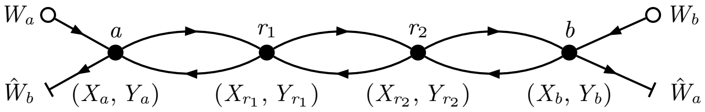

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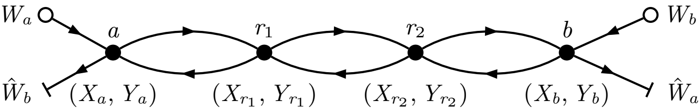

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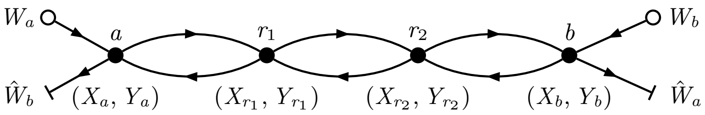

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- Transmission strategy: physical-layer network coding with nested Voronoi codes

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- No noise, binary inputs and outputs, finite field physical layer, full-duplex nodes

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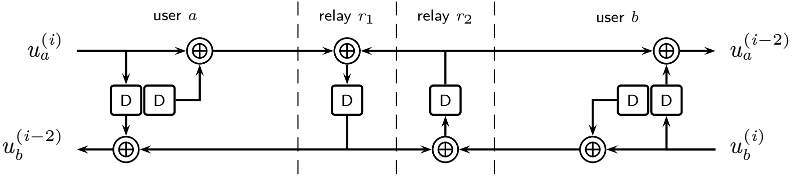

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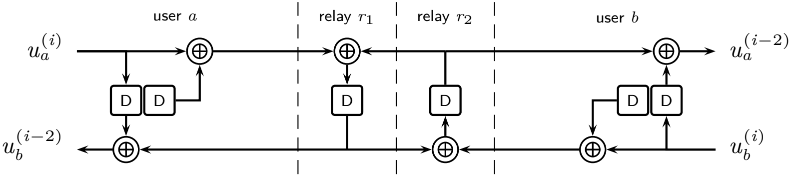

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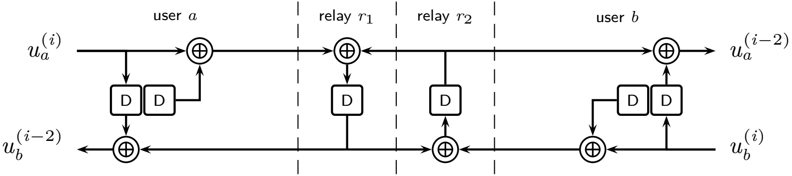

- Each user “sees” the packet stream of the other user – delayed by two blocks.
- User rates approach 1 bit per channel use for many blocks.

The Gaussian Case

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Channel model:

$$\begin{aligned} Y_a &= X_{r_1} + Z_a \\ Y_{r_1} &= X_a + X_{r_2} + Z_{r_1} \\ Y_{r_2} &= X_{r_1} + X_b + Z_{r_2} \\ Y_b &= X_{r_2} + Z_b \end{aligned}$$

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Theorem

One can achieve all rate pairs (R_a, R_b) *satisfying*

$$R_a < \min\left(\tilde{\mathrm{C}}\left(P/\sigma_{r_1}^2\right), \tilde{\mathrm{C}}\left(P/\sigma_{r_2}^2\right), \mathrm{C}\left(P/\sigma_b^2\right)\right)$$
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MLAN Conversion

See [Erez-Zamir '08].

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Blockwise modulo-lattice additive noise channel:

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\tilde{\boldsymbol{Y}}_a^{(i)} &= (\tilde{\boldsymbol{X}}_{r_1}^{(i)} + \tilde{\boldsymbol{Z}}_a^{(i)}) \mod \Lambda \\
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where

- $\tilde{\boldsymbol{X}}_a^{(i)}, \tilde{\boldsymbol{X}}_{r_1}^{(i)}, \tilde{\boldsymbol{X}}_{r_2}^{(i)}, \tilde{\boldsymbol{X}}_b^{(i)} \in \mathcal{R}_V(\Lambda)$ are the input signals in block i.

MLAN Conversion

See [Erez-Zamir '08].

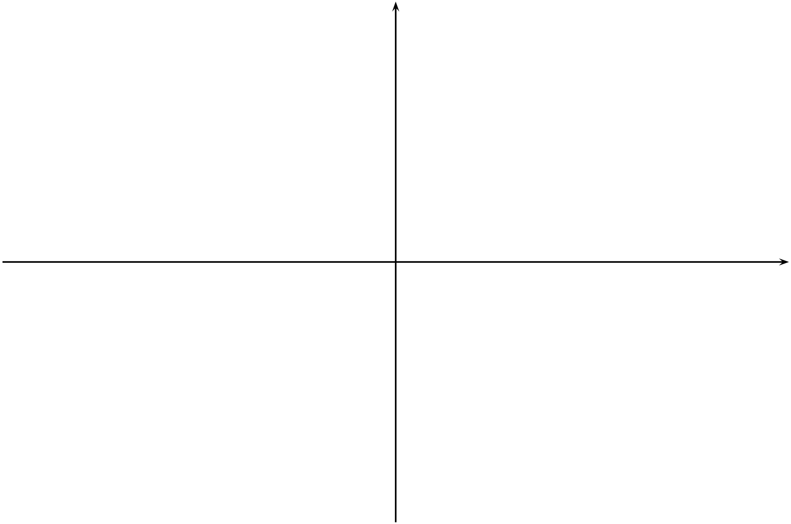
Blockwise modulo-lattice additive noise channel:

$$\begin{aligned}
\tilde{\boldsymbol{Y}}_a^{(i)} &= (\tilde{\boldsymbol{X}}_{r_1}^{(i)} + \tilde{\boldsymbol{Z}}_a^{(i)}) \mod \Lambda \\
\tilde{\boldsymbol{Y}}_{r_1}^{(i)} &= (\tilde{\boldsymbol{X}}_a^{(i)} + \tilde{\boldsymbol{X}}_{r_2}^{(i)} + \tilde{\boldsymbol{Z}}_{r_1}^{(i)}) \mod \Lambda \\
\tilde{\boldsymbol{Y}}_{r_2}^{(i)} &= (\tilde{\boldsymbol{X}}_{r_1}^{(i)} + \tilde{\boldsymbol{X}}_b^{(i)} + \tilde{\boldsymbol{Z}}_{r_2}^{(i)}) \mod \Lambda \\
\tilde{\boldsymbol{Y}}_b^{(i)} &= (\tilde{\boldsymbol{X}}_{r_2}^{(i)} + \tilde{\boldsymbol{Z}}_b^{(i)}) \mod \Lambda
\end{aligned}$$

where

- $\tilde{\boldsymbol{X}}_a^{(i)}, \tilde{\boldsymbol{X}}_{r_1}^{(i)}, \tilde{\boldsymbol{X}}_{r_2}^{(i)}, \tilde{\boldsymbol{X}}_b^{(i)} \in \mathcal{R}_V(\Lambda)$ are the input signals in block i.
- Effective noise $\tilde{\boldsymbol{Z}}_a^{(i)}, \tilde{\boldsymbol{Z}}_{r_1}^{(i)}, \tilde{\boldsymbol{Z}}_{r_2}^{(i)}, \tilde{\boldsymbol{Z}}_b^{(i)}$ is statistically independent of the inputs and has reduced effective noise power.

Nested Voronoi Codes



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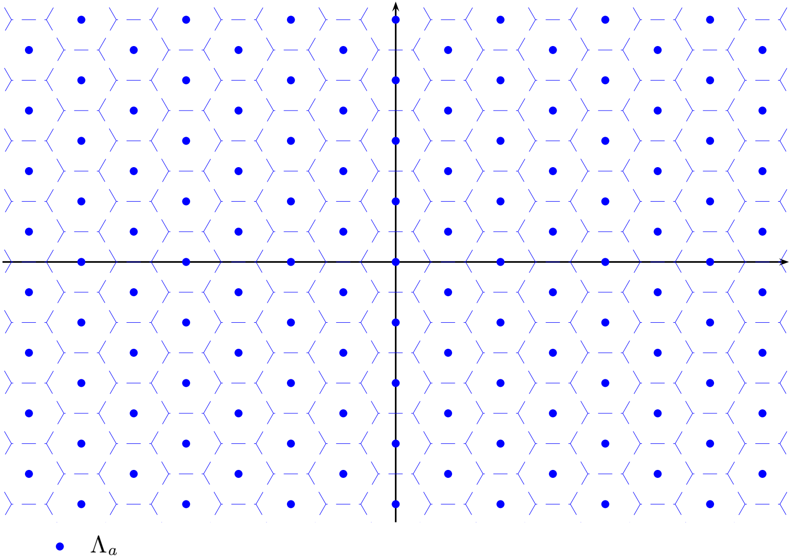

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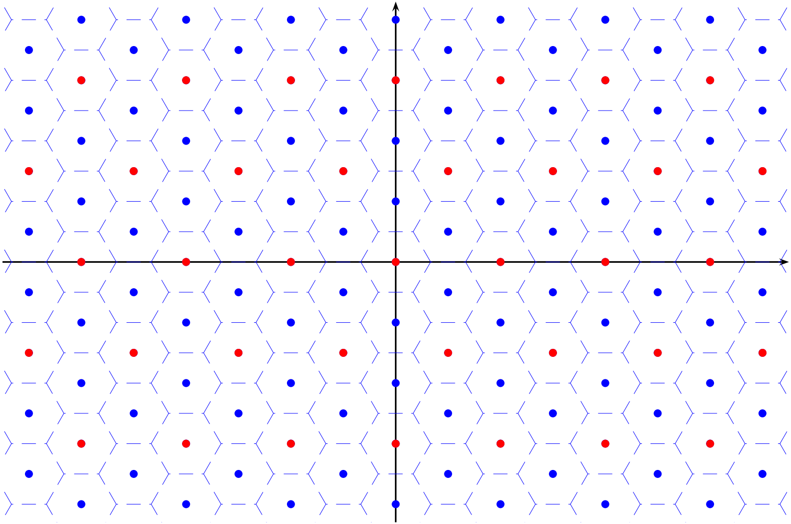

●● Λ_a ● Λ_b

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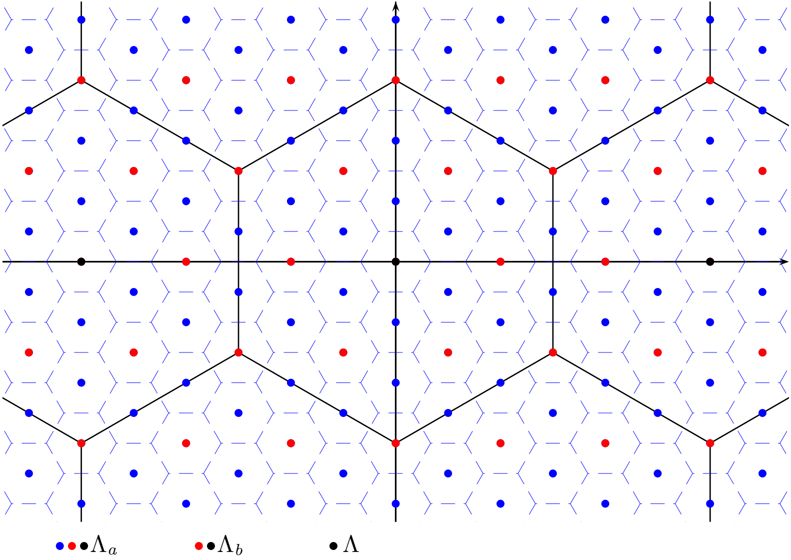

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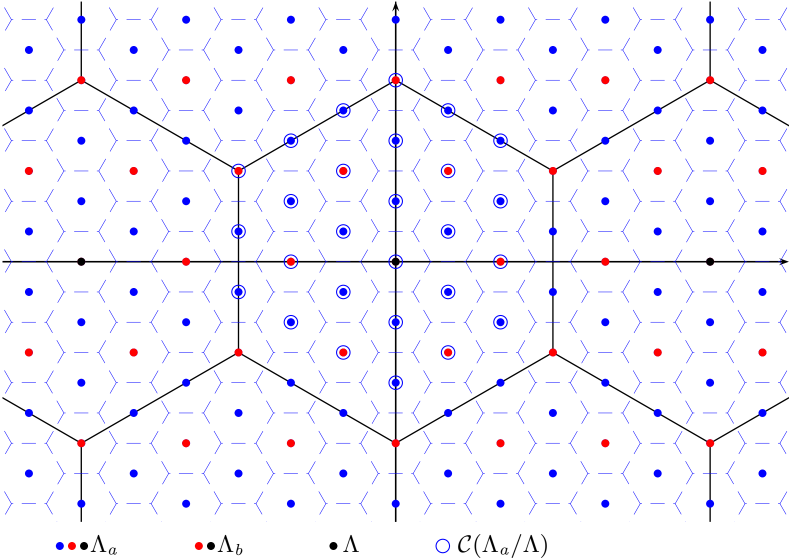
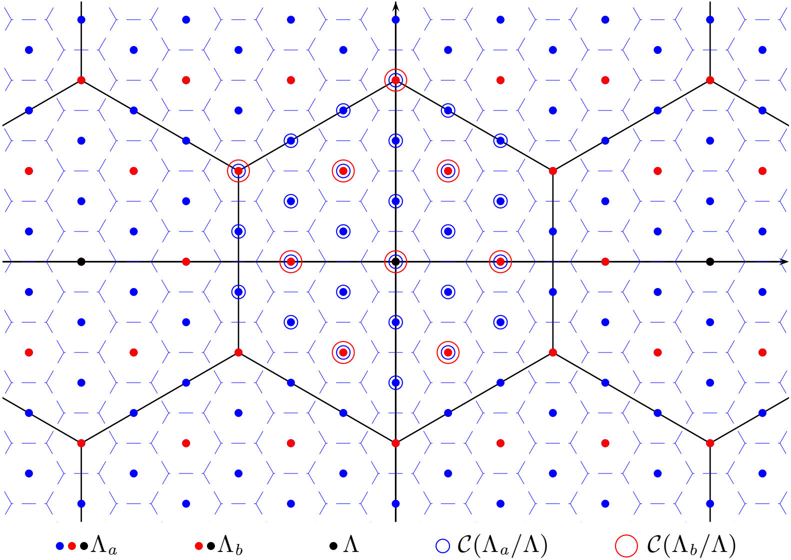
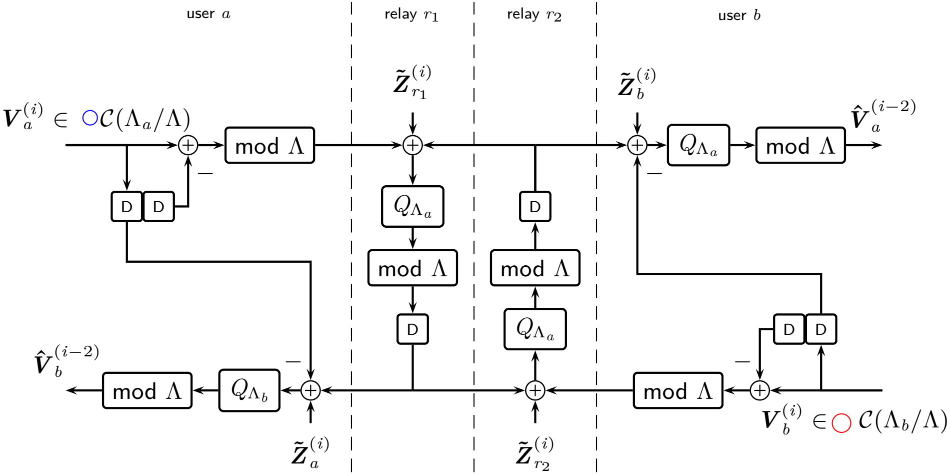


CHALMERS

Nested Voronoi Codes



Block Diagram



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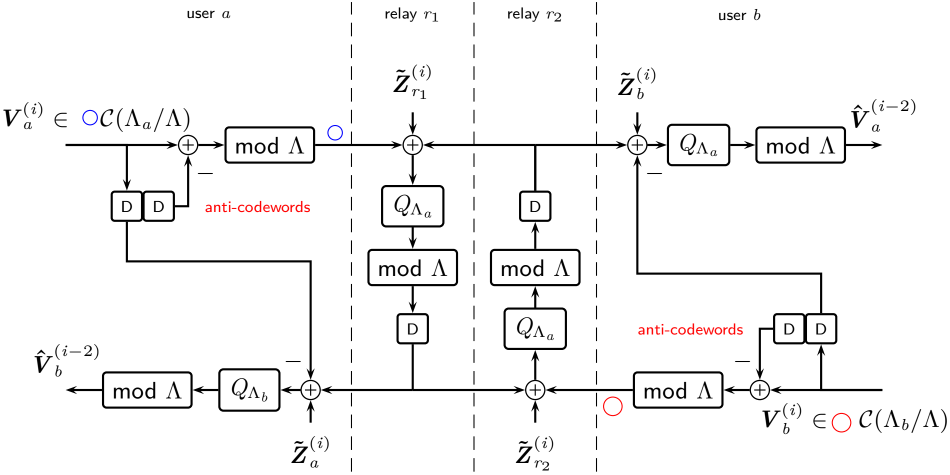
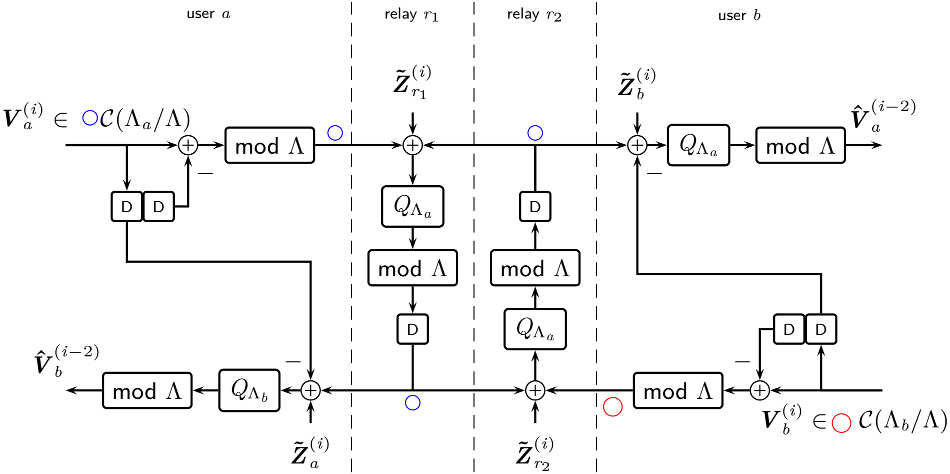
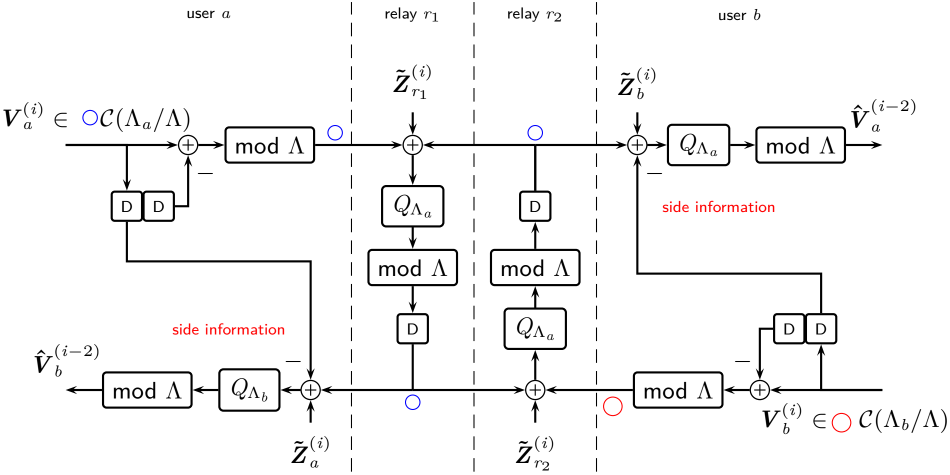


Block Diagram



relays "protect" linear combinations of codewords

Block Diagram



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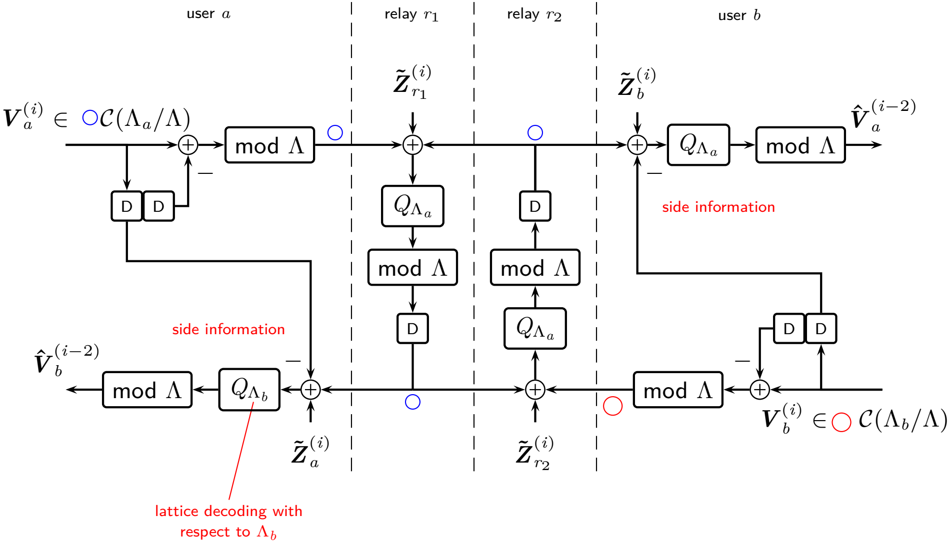

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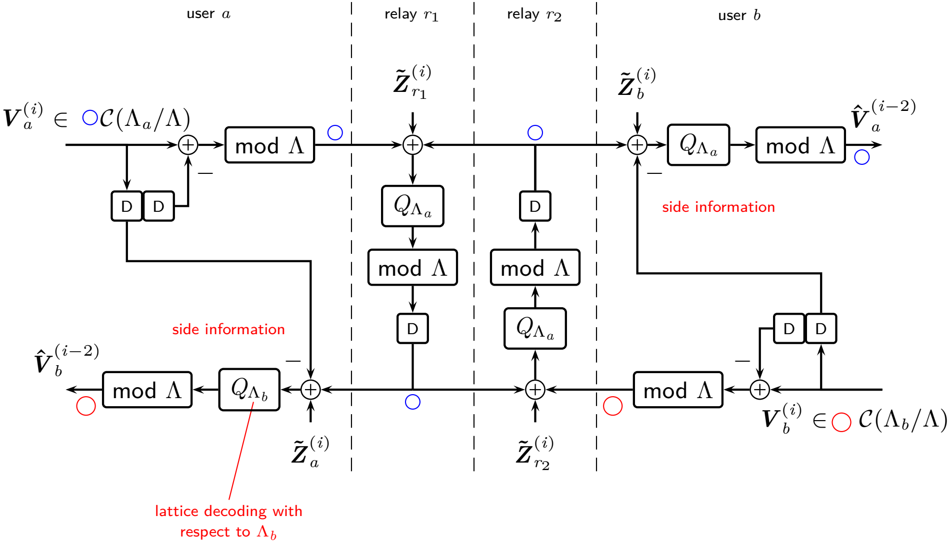

Final Remarks

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- Different shaping lattices might provide a solution.
- Generalization to L relays (assuming full separation) is possible.

Conclusion

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- Structured codes appear to be a powerful tool to show achievability of rates in communication networks.
- Involved principles: physical-layer network coding, broadcasting, using side information
- In certain networks the interference of users can be harnessed.
- However, a strategy to show full achievability of the upper bound is still not available.

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Downlink

	1	$\cdots$	$2^{n(R_a - R_b)}$
1	$\boldsymbol{X}_r(1,1)$	$\cdots$	$\boldsymbol{X}_r(1,M)$
2	$\boldsymbol{X}_r(2,1)$	$\cdots$	$\boldsymbol{X}_r(2,M)$
$\vdots$	$\vdots$		$\vdots$
2^{nR_b}	$\boldsymbol{X}_r(N,1)$	$\cdots$	$\boldsymbol{X}_r(N,M)$

Downlink

	1	$\cdots$	$2^{n(R_a-R_b)}$
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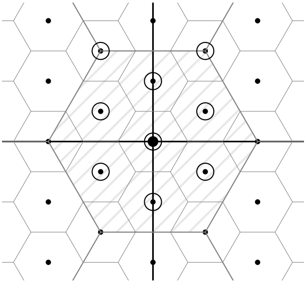
- Random codebook: $N = 2^{nR_b}$ rows, $M = 2^{n(R_a-R_b)}$ columns

Downlink

$\boldsymbol{V} \in \mathcal{C}(\Lambda_c/\Lambda)$



	1	$\cdots$	$2^{n(R_a-R_b)}$
1	$\boldsymbol{X}_r(1,1)$	$\cdots$	$\boldsymbol{X}_r(1,M)$
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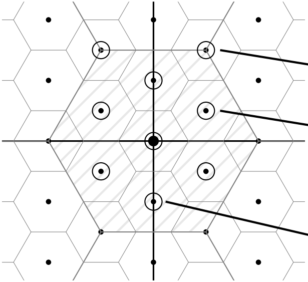

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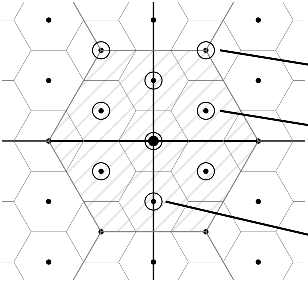

extra message

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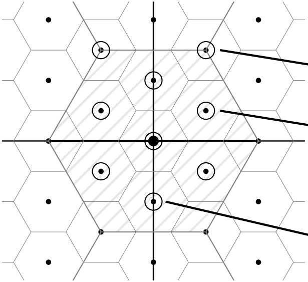

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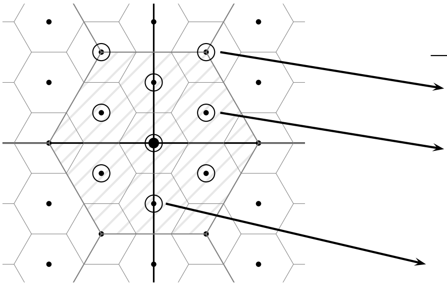

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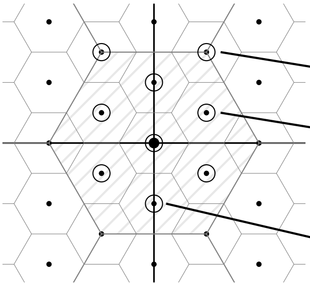

extra message

	1	$\cdots$	$2^{n(R_a-R_b)}$
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- User b decodes with respect to the whole codebook.
- User a decodes with respect to a column of the codebook.
- Optimal downlink strategy.

Another Toy Problem

- Noiseless, finite field physical layer and half-duplex nodes
- Users exchange bits (packets) $u_a^{(1)}, u_a^{(2)}, \ldots$ and $u_b^{(1)}, u_b^{(2)}, \ldots$

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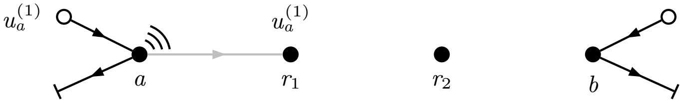

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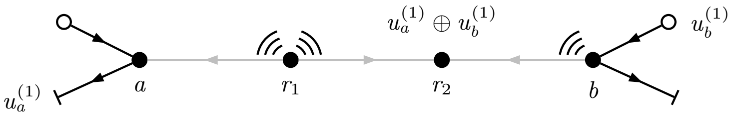

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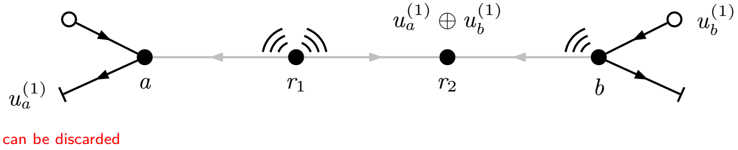

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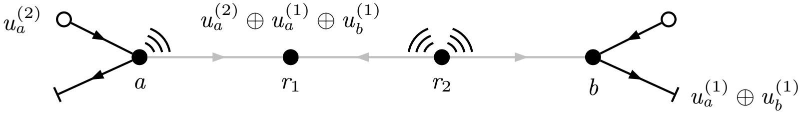

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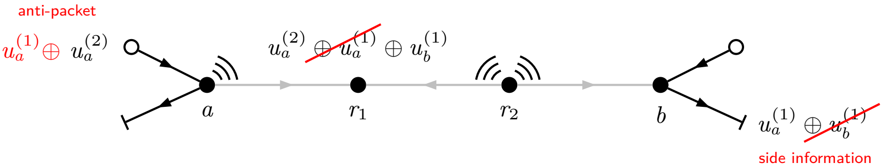

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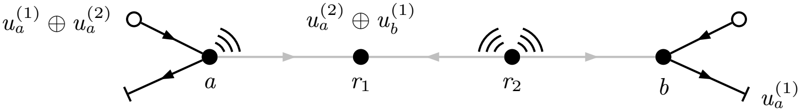

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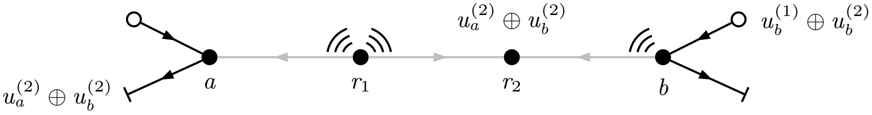

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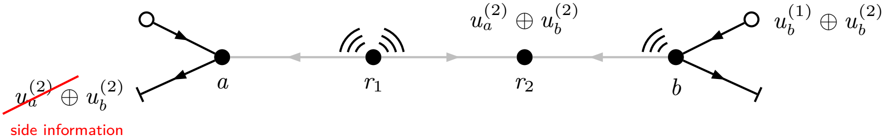

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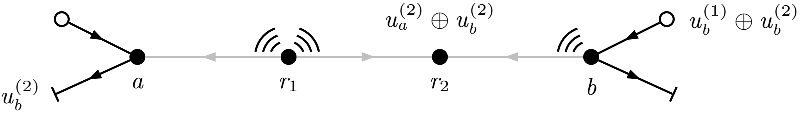

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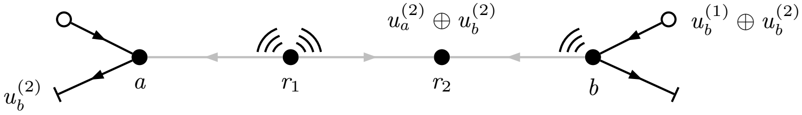

- Steady state

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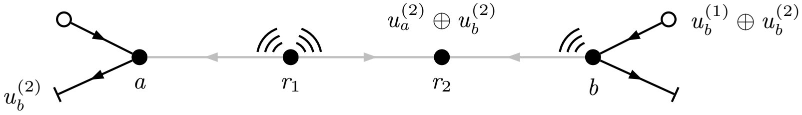

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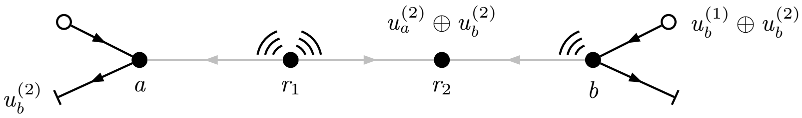

- Steady state
- Both users send a new packet or receive a packet in each block
- Sum rate approaches 2 bits per 2 transmissions

Full-Duplex Block Transmission

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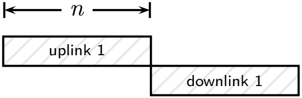
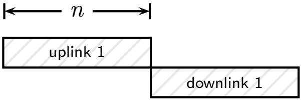


Two phases:

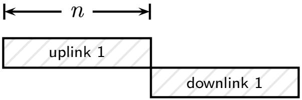
Full-Duplex Block Transmission



Two phases:

1. Uplink: users a and b transmit to the relay r.

Full-Duplex Block Transmission



Two phases:

1. Uplink: users a and b transmit to the relay r.
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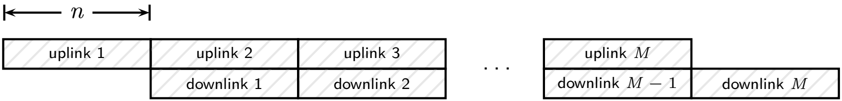

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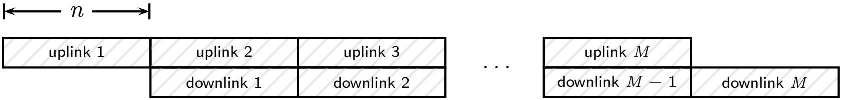

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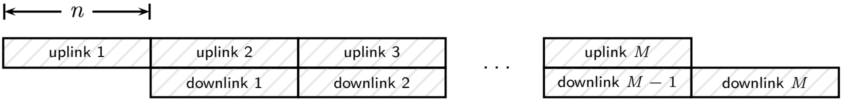
Choose number of blocks M (uplink-downlink pairs) large, such that

$$\frac{MnR_a}{(M+1)n} \approx R_a, \qquad \frac{MnR_b}{(M+1)n} \approx R_b.$$

Full-Duplex Block Transmission



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What rate pairs (R_a, R_b) (in bits per channel use) are achievable with $p_e \to 0$ as $n \to \infty$?