Performance Analysis and Anchor Decoding of Staircase Codes

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Joint work with: Henry D. Pfister[2], Alexandre Graell i Amat[1], Fredrik Brännström[1], and Erik Agrell[1]

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CHALMERS

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FIBER-OPTIC COMMUNICATIONS
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Outline

1. Introduction: Forward-Error Correction for Fiber-Optic Systems
2. Generalized Product Codes and Iterative Decoding
3. Asymptotic Performance Analysis with Density Evolution
4. Avoiding Miscorrections via Anchor Decoding
5. Conclusions

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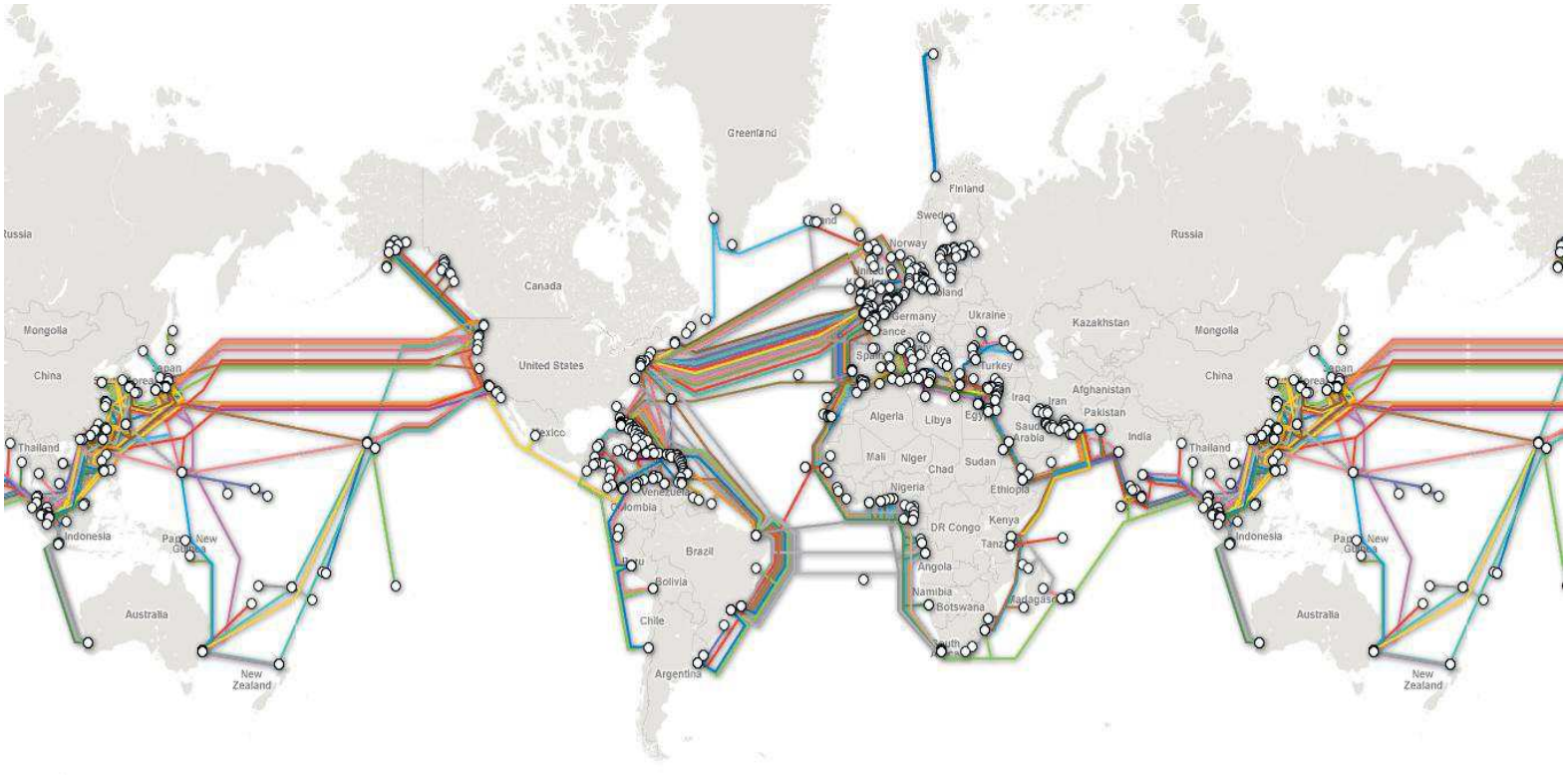
Fiber-Optic Communications



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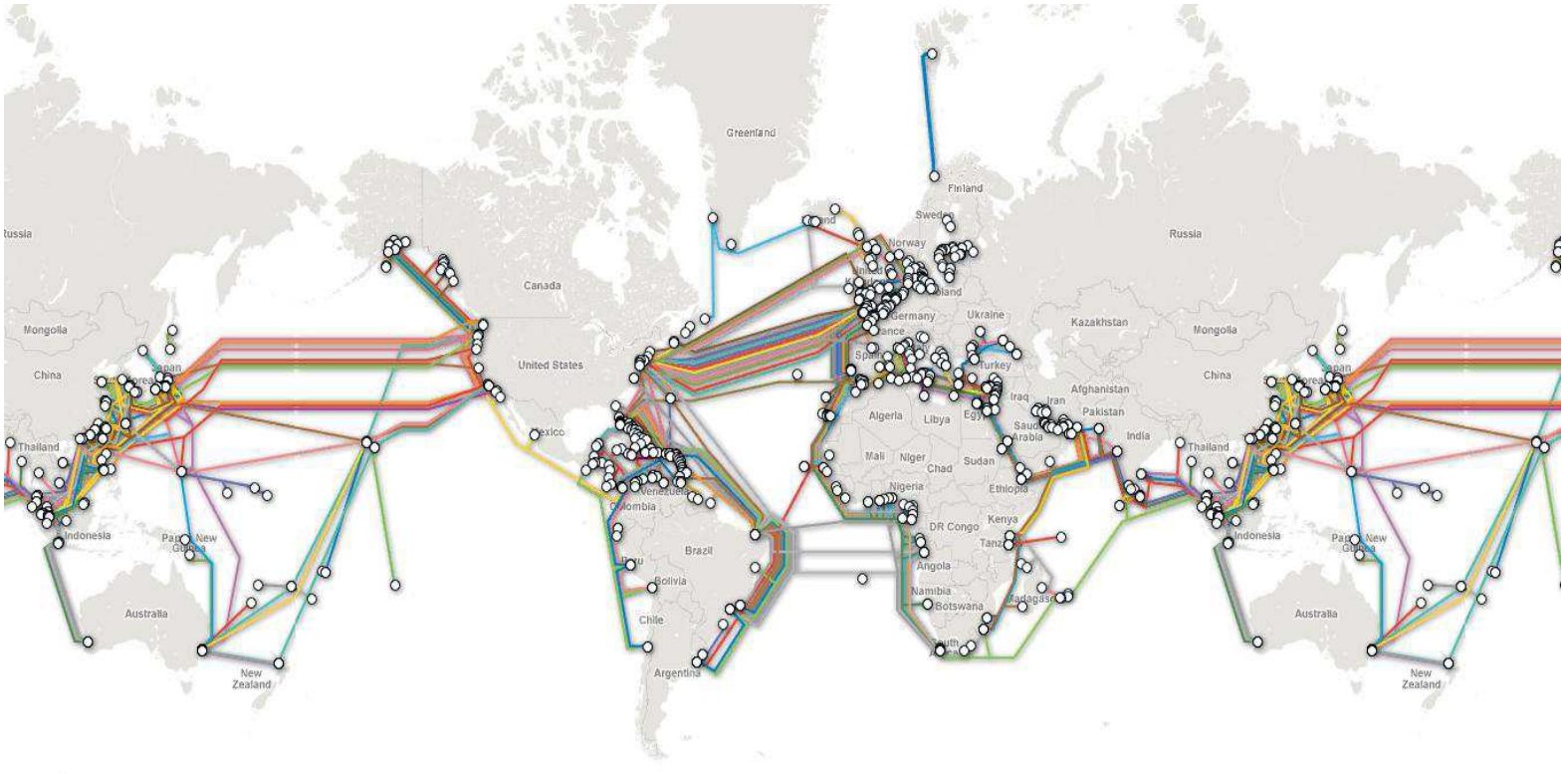

Fiber-optic communication systems enable data traffic over very long distances connecting cities, countries, and continents.

Fiber-Optic Communications

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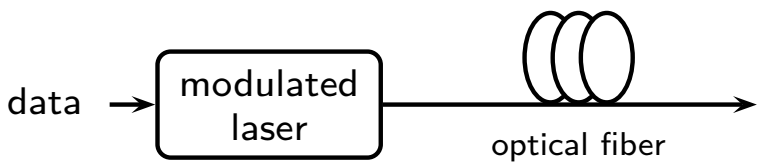

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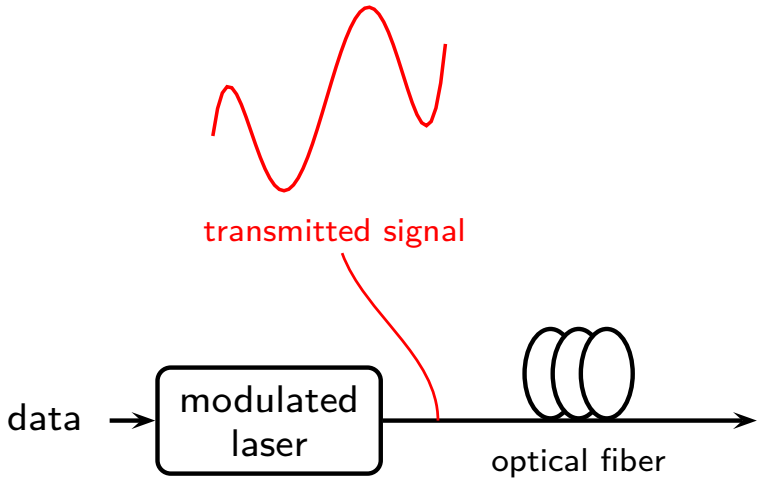

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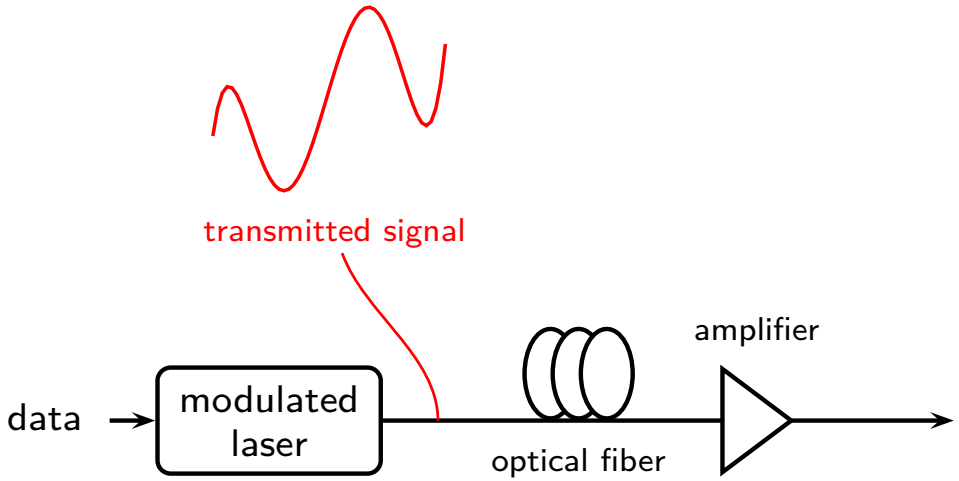

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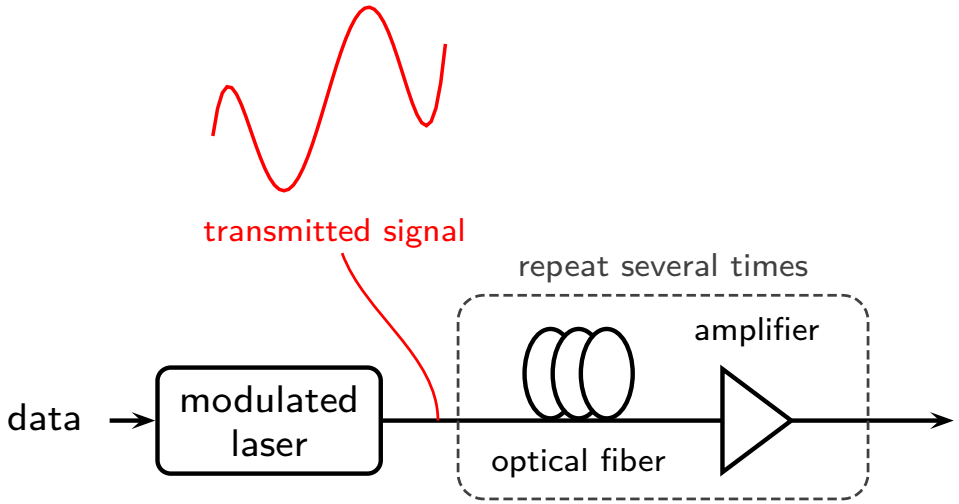

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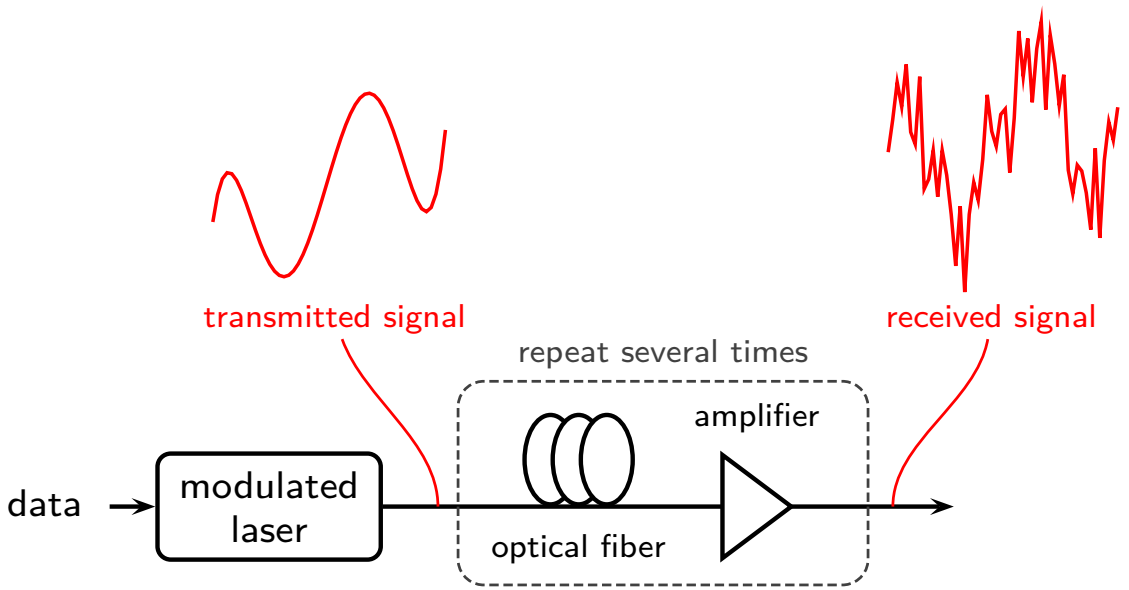

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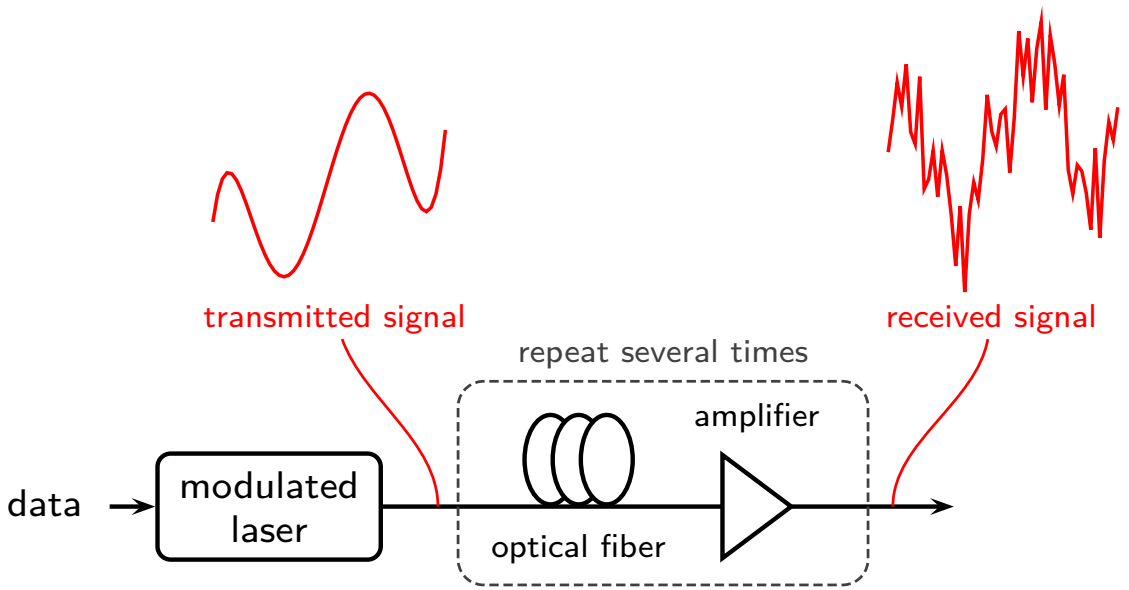
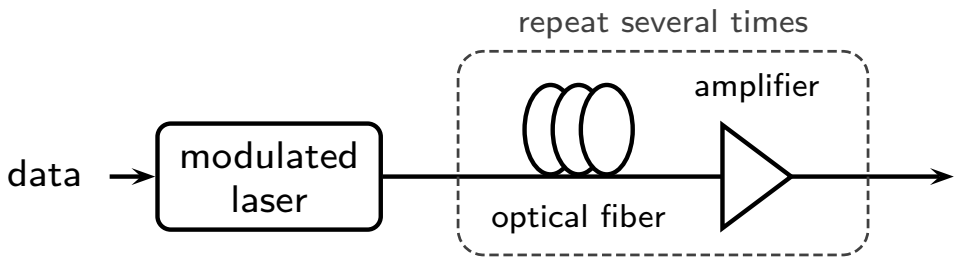
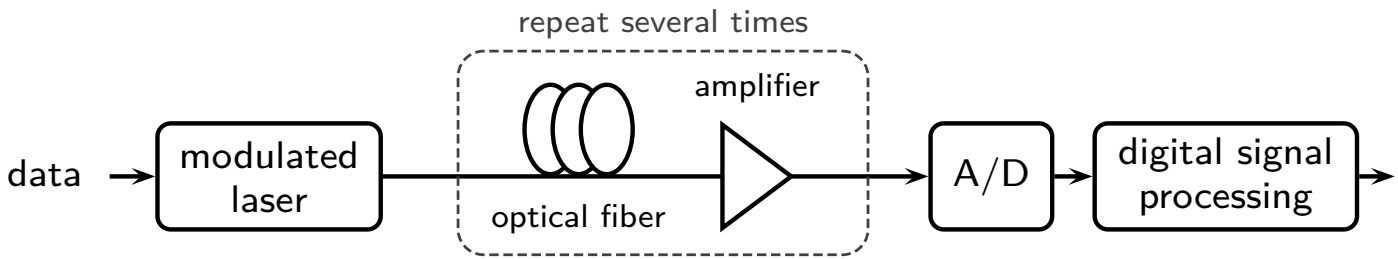


- Long distances result in significant signal attenuation
- Periodic amplification necessary $\Longrightarrow$ random distortions or noise
- Error-correcting codes ensure reliable data transmission

Fiber-Optic Communications



Error-Correcting Codes



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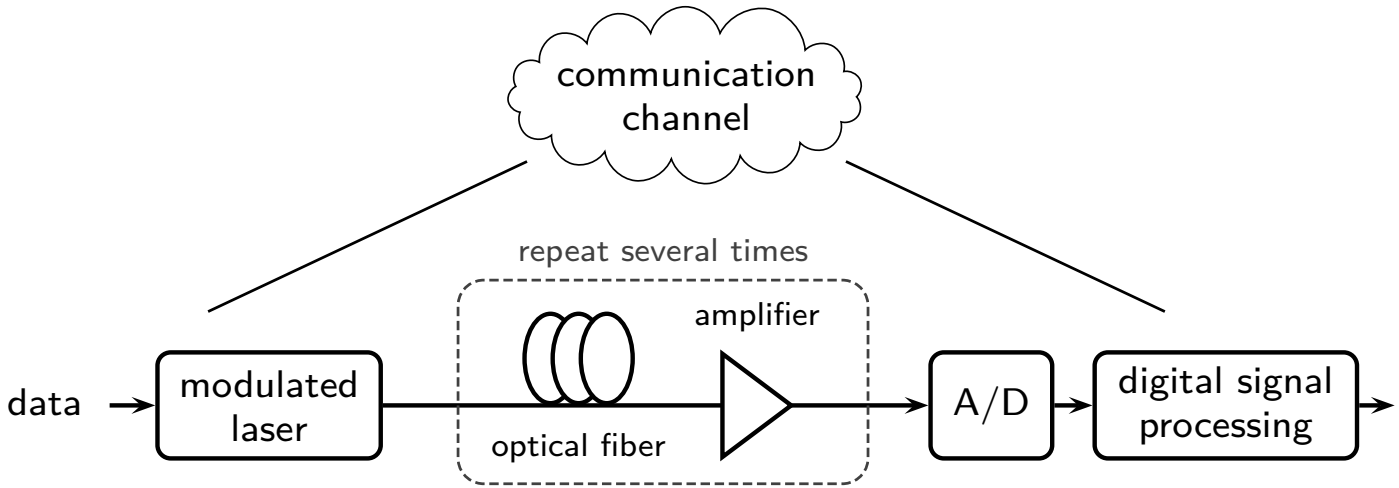

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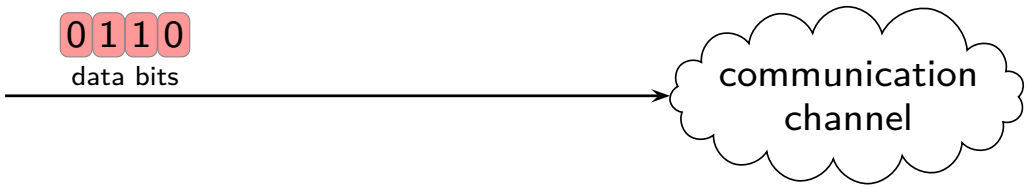

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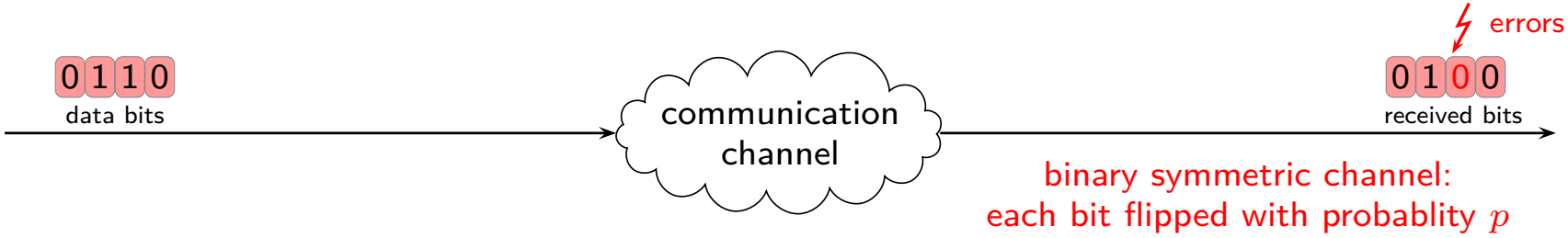

binary symmetric channel:
each bit flipped with probablity p

(Motivation: limited soft-information in metro networks, outer clean-up codes, . . .)

Error-Correcting Codes

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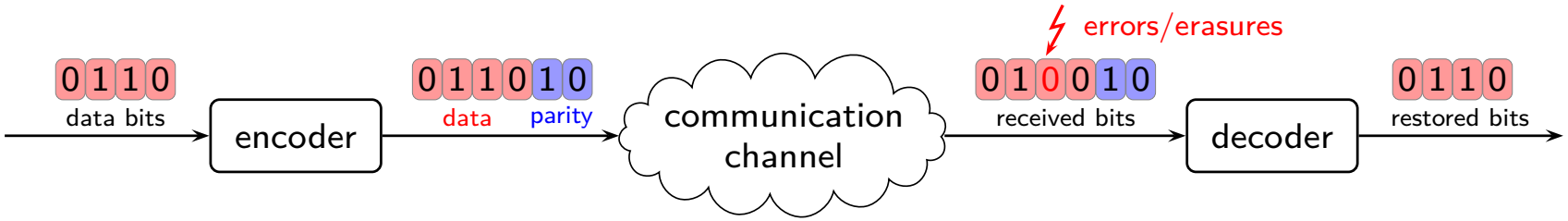

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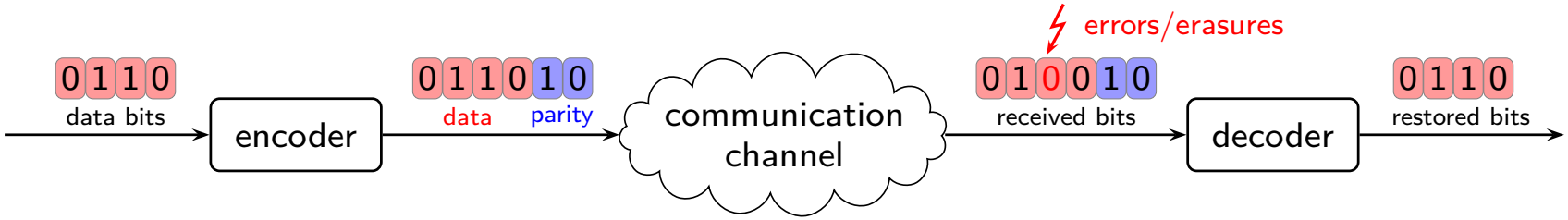

Requirements for Fiber-Optic Communications

- Very high throughputs (100 Gigabits per second or higher)
- Very high net coding gains (close-to-capacity performance)
- Very low bit error rates (below 10^{-15})

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This talk

1. Asymptotic performance of deterministic generalized product codes
2. Binary erasure channel vs. binary symmetric channel

Outline

Product Codes and Staircase Codes

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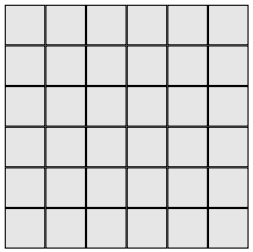
rectangular array [Elias, 1954]



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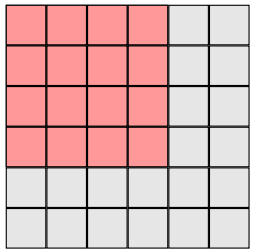

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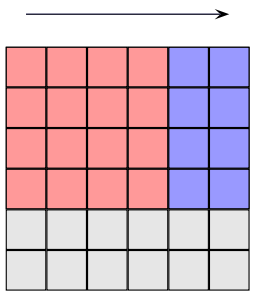

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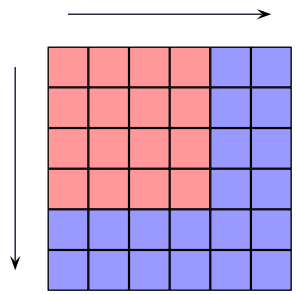

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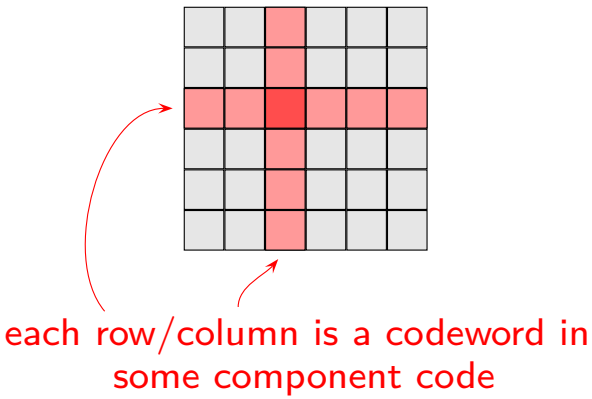

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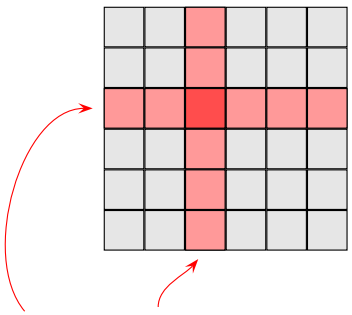
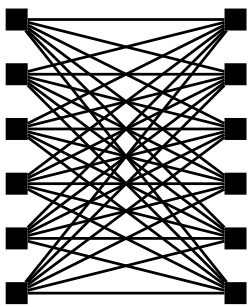


each row/column is a codeword in some component code

Tanner graph



constraint node degree = component code length

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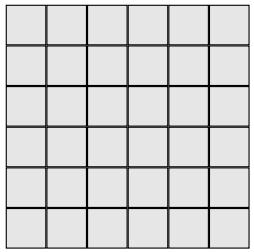

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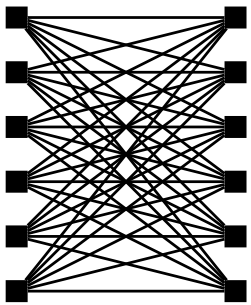

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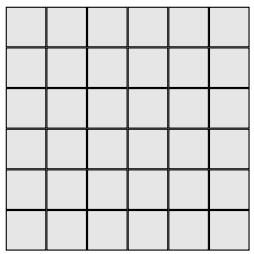

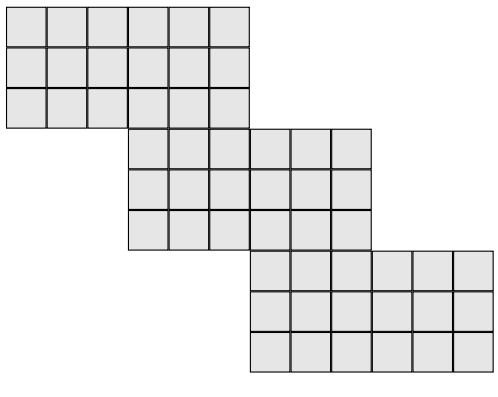

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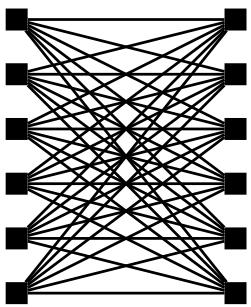

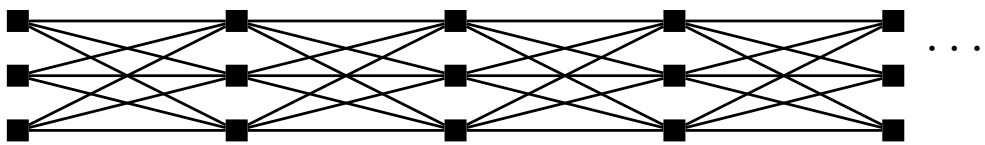

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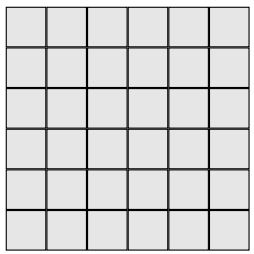

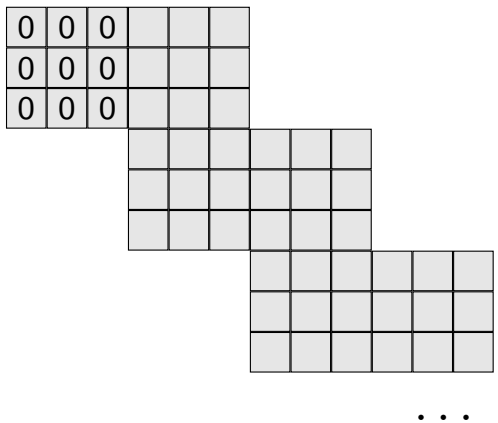

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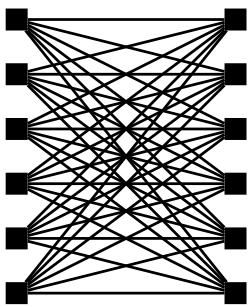

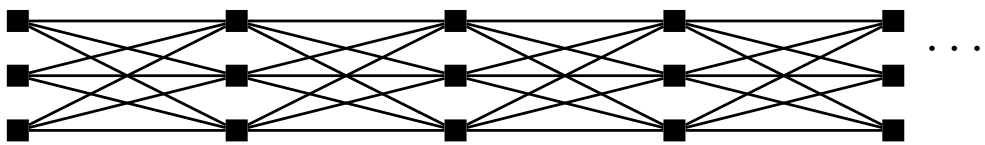

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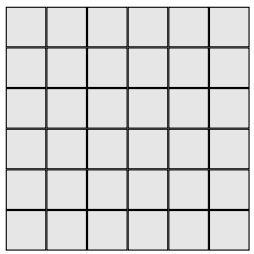

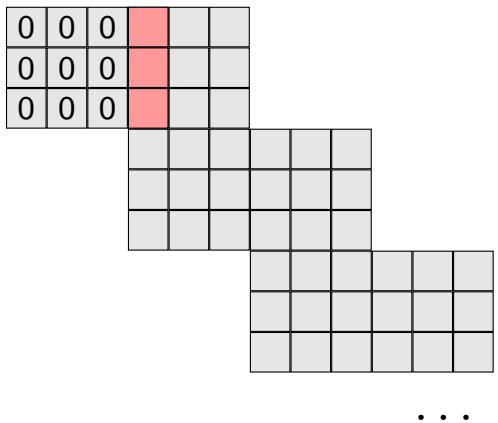

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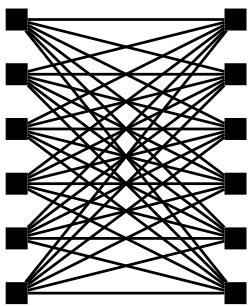

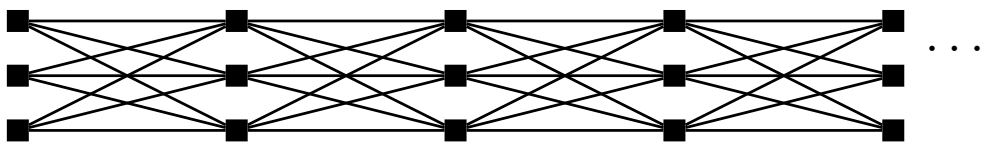

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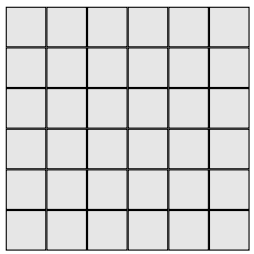

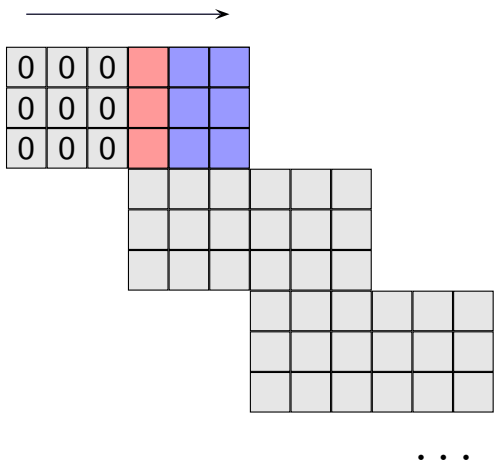

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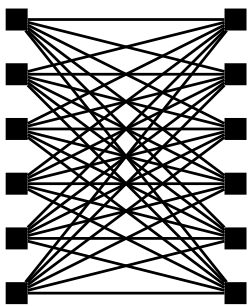

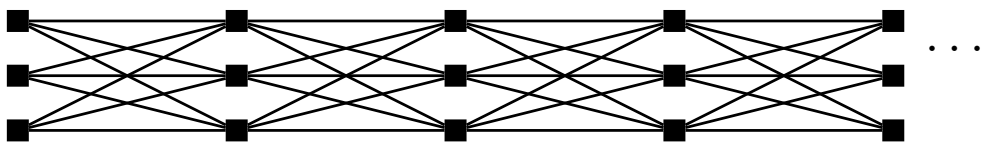

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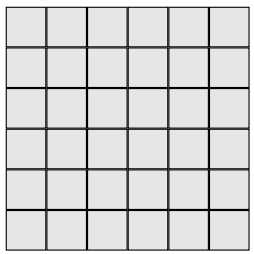

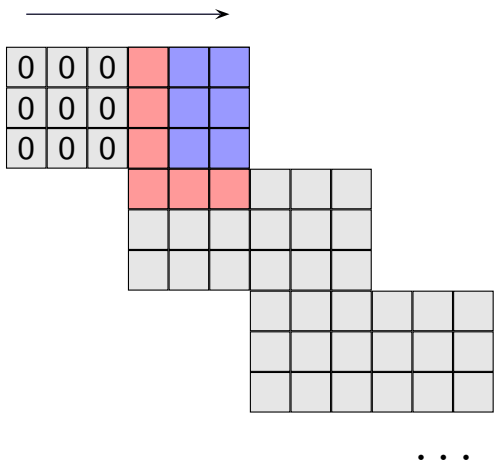

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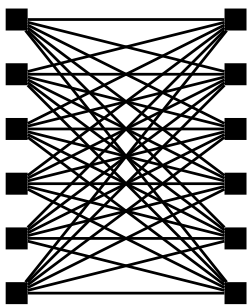

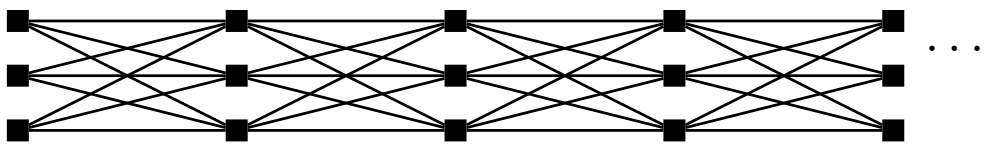

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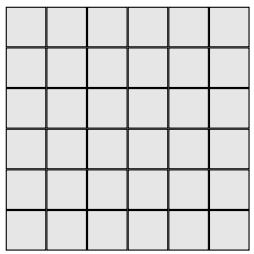

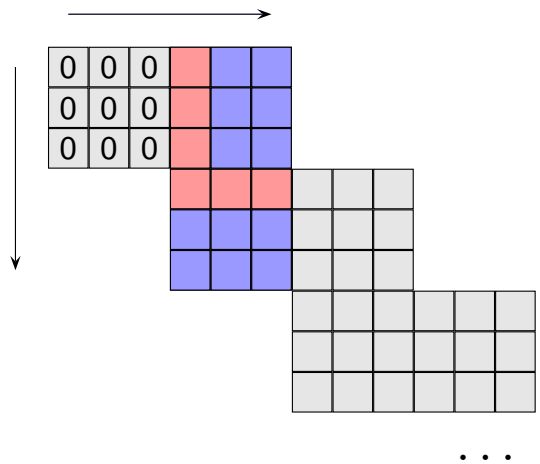

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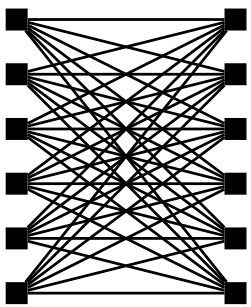

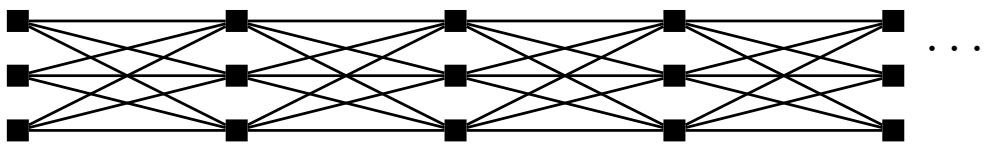

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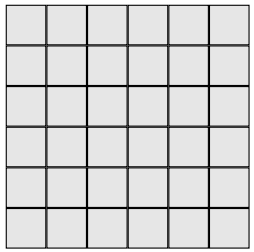

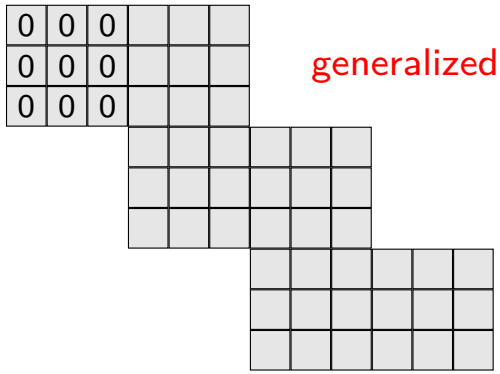

generalized product code (GPC)

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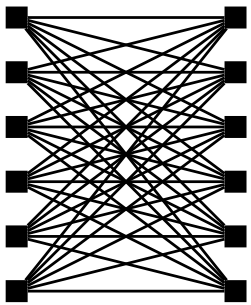

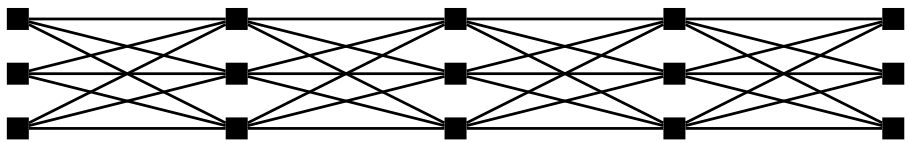

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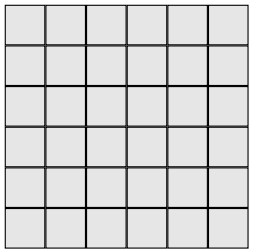
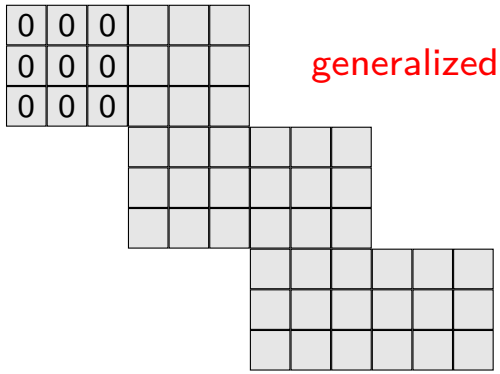
rectangular array [Elias, 1954]

staircase array [Smith et al., 2012]

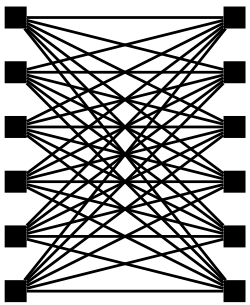


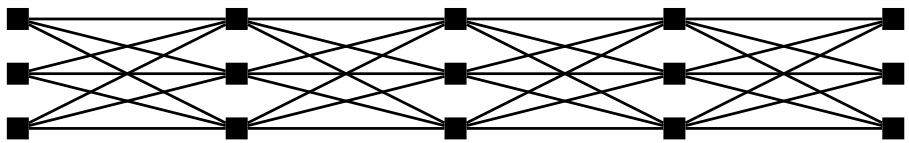


generalized product code (GPC)

. . .

Tanner graph

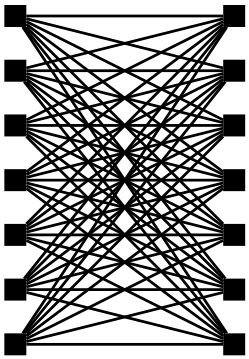


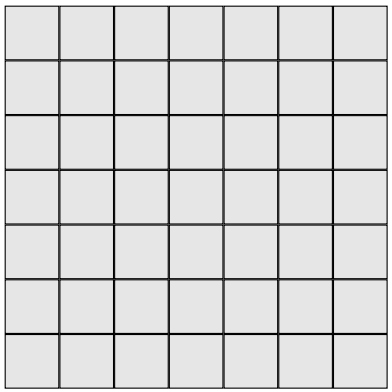


. . .

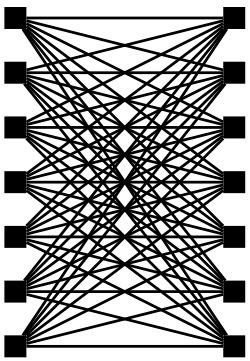
- Deterministic codes with fixed and structured Tanner graph

Iterative Bounded-Distance Decoding



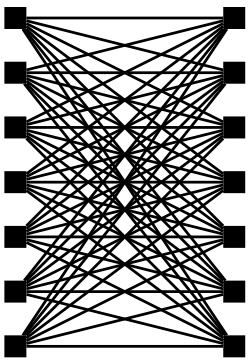


Iterative Bounded-Distance Decoding



0	1	0	1	0	1	0
0	1	0	1	1	0	1
0	1	0	1	0	1	0
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	0	0	1	1	1
0	1	0	0	0	1	1

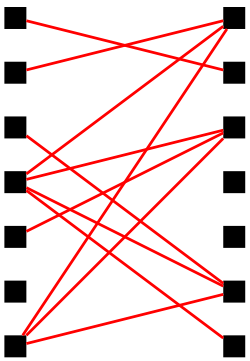
Iterative Bounded-Distance Decoding



0	?	0	?	0	1	?
?	1	0	1	1	0	1
0	1	0	?	0	?	?
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	?	?	1	1	?
0	1	0	?	0	1	1

- Codeword transmission over binary erasure channel with erasure probability p

Iterative Bounded-Distance Decoding

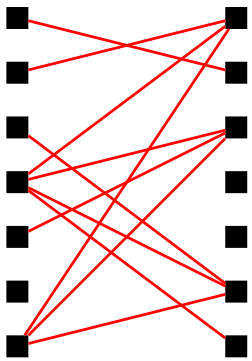


residual graph

0	?	0	?	0	1	?
?	1	0	1	1	0	1
0	1	0	?	0	?	?
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	?	?	1	1	?
0	1	0	?	0	1	1

- Codeword transmission over binary erasure channel with erasure probability p

Iterative Bounded-Distance Decoding

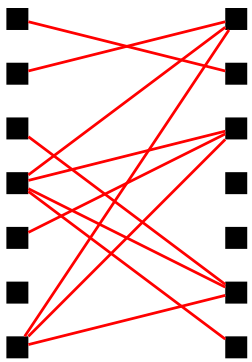


residual graph

0	?	0	?	0	1	?
?	1	0	1	1	0	1
0	1	0	?	0	?	?
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	?	?	1	1	?
0	1	0	?	0	1	1

- Codeword transmission over binary erasure channel with erasure probability p
- Each component code corrects $\leq t$ erasures

Iterative Bounded-Distance Decoding



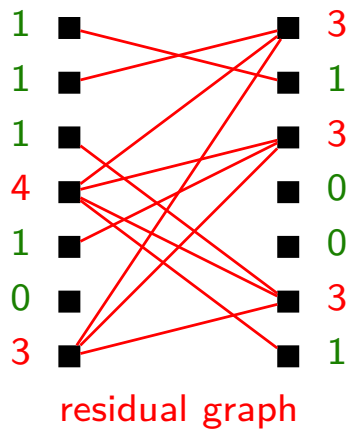
residual graph

0	?	0	?	0	1	?
?	1	0	1	1	0	1
0	1	0	?	0	?	?
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	?	?	1	1	?
0	1	0	?	0	1	1

- Codeword transmission over binary erasure channel with erasure probability p
- Each component code corrects $\leq t$ erasures
- ℓ iterations of bounded-distance decoding $=$ peeling of vertices with degree $\leq t$ (in parallel)

Iterative Bounded-Distance Decoding

1st iteration $(t = 2)$

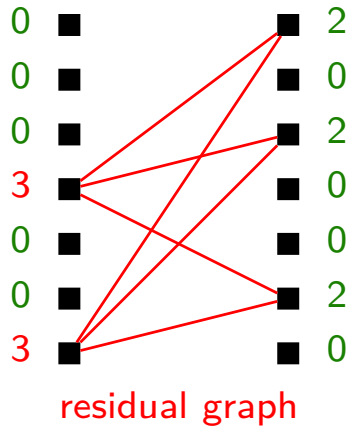


0	?	0	?	0	1	?
?	1	0	1	1	0	1
0	1	0	?	0	?	?
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	?	?	1	1	?
0	1	0	?	0	1	1

- Codeword transmission over binary erasure channel with erasure probability p
- Each component code corrects $\leq t$ erasures
- ℓ iterations of bounded-distance decoding $=$ peeling of vertices with degree $\leq t$ (in parallel)

Iterative Bounded-Distance Decoding

2nd iteration $(t = 2)$

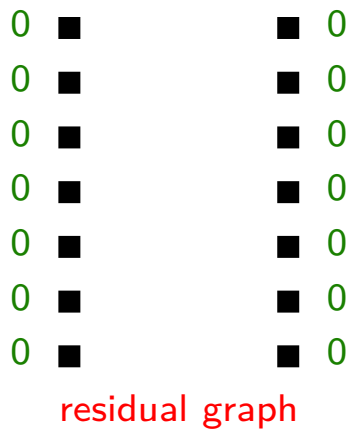


0	1	0	?	0	1	?
0	1	0	1	1	0	1
0	1	0	?	0	1	?
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	0	?	1	1	?
0	1	0	0	0	1	1

- Codeword transmission over binary erasure channel with erasure probability p
- Each component code corrects $\leq t$ erasures
- ℓ iterations of bounded-distance decoding $=$ peeling of vertices with degree $\leq t$ (in parallel)

Iterative Bounded-Distance Decoding

2nd iteration $(t = 2)$



0	1	0	1	0	1	0
0	1	0	1	1	0	1
0	1	0	1	0	1	0
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	0	0	1	1	1
0	1	0	0	0	1	1

- Codeword transmission over binary erasure channel with erasure probability p
- Each component code corrects $\leq t$ erasures
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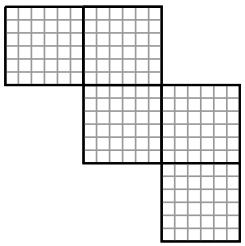
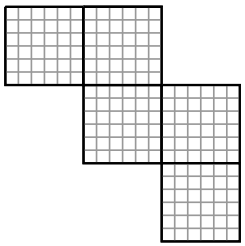
Outline

Performance Prediction

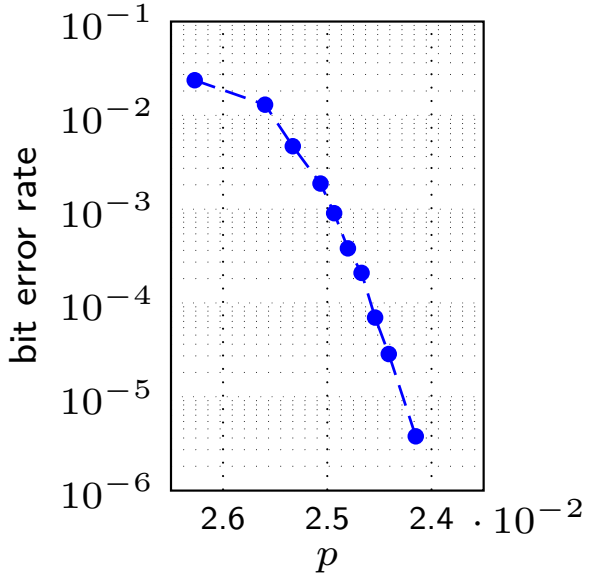


- Example: staircase code with a fixed component code

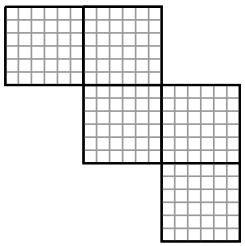
Performance Prediction

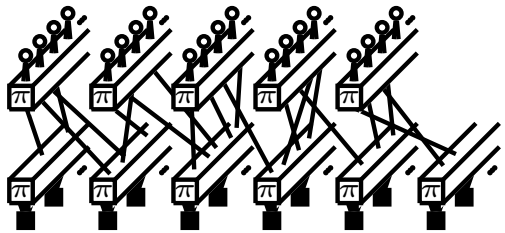


- Example: staircase code with a fixed component code
- Use simulations to predict performance → computationally intensive



Performance Prediction





- Example: staircase code with a fixed component code
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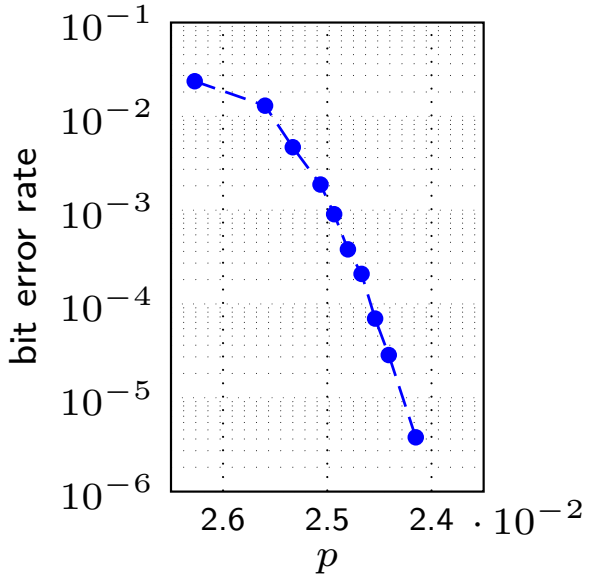
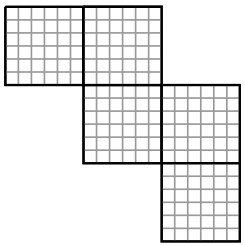
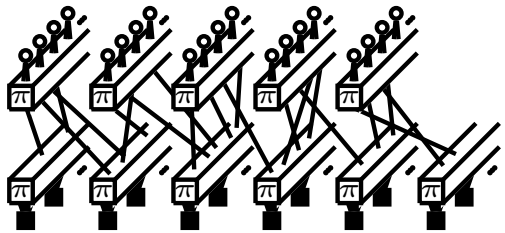
- Define randomized set of generalized product codes [Jian et al., 2012], [Zhang et al., 2015]



Performance Prediction





- Example: staircase code with a fixed component code
- Use simulations to predict performance → computationally intensive
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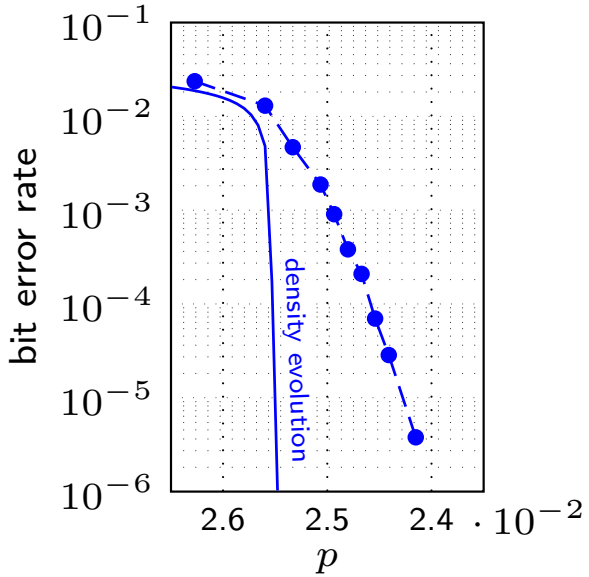
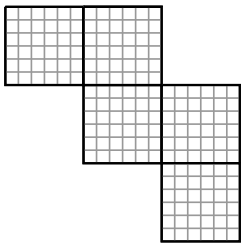
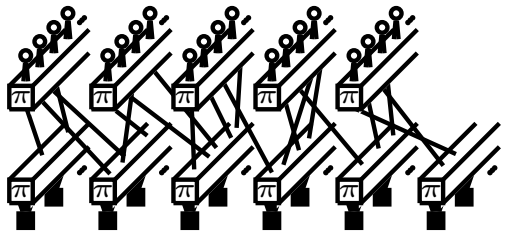


Performance Prediction





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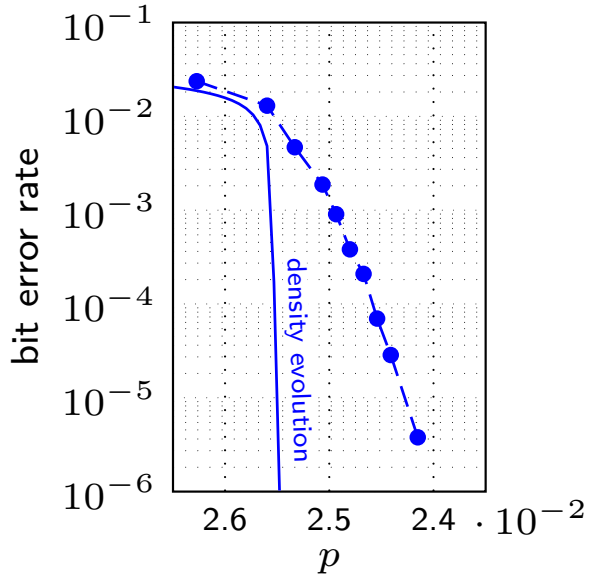
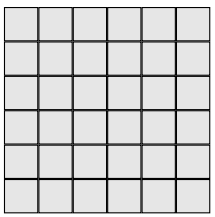
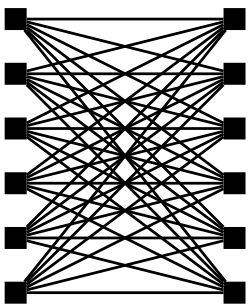


Is it possible to directly analyze deterministic generalized product codes?

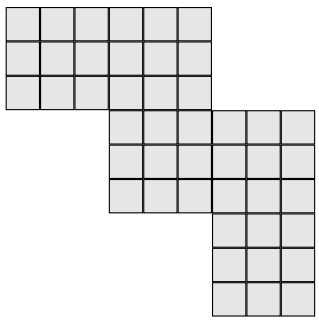
Parameterized Construction of Generalized Product Codes

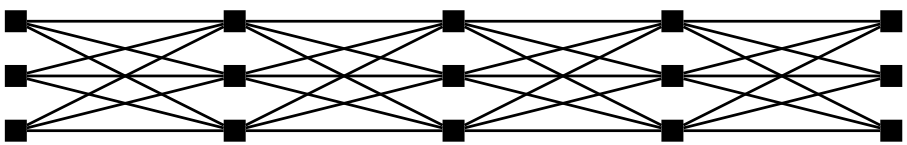
product codes





staircase codes

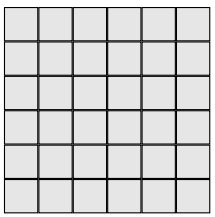


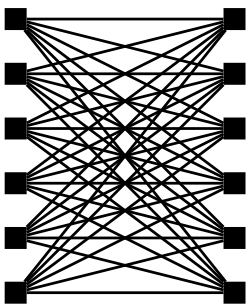


positions: 1 2 3 4 5

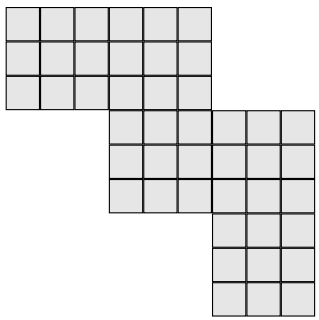
Parameterized Construction of Generalized Product Codes

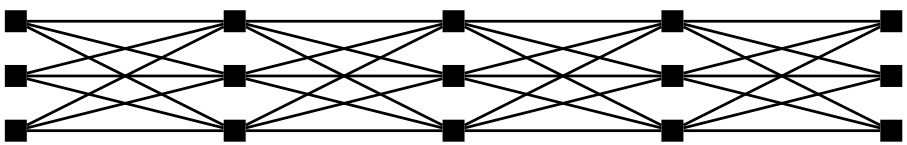
product codes





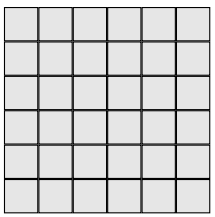
staircase codes

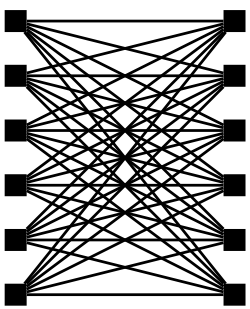




Parameterized Construction of Generalized Product Codes

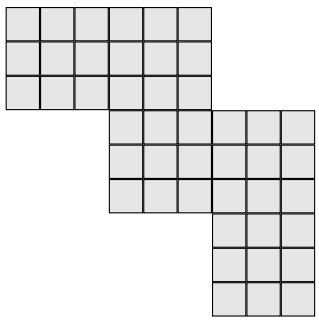
product codes



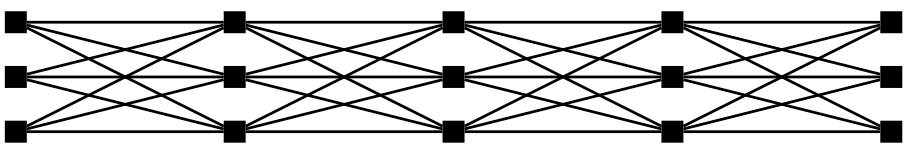


1 $L = 2$

staircase codes

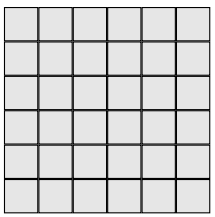


positions: 1 2 3 4 $L = 5$

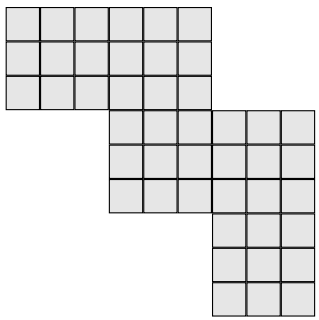


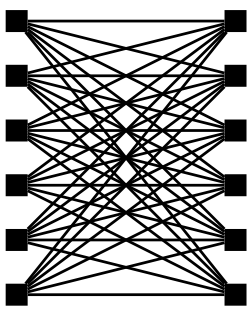
Parameterized Construction of Generalized Product Codes

product codes



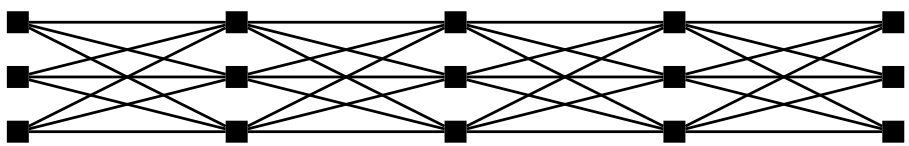
staircase codes





1 $L = 2$

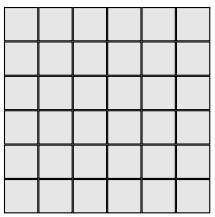
positions: 1 2 3 4 $L = 5$



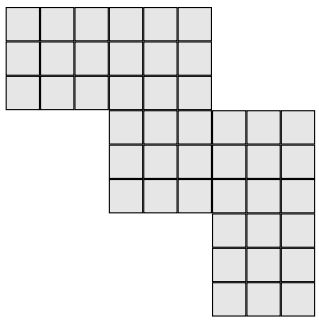
$\boldsymbol{\eta}$: symmetric $L \times L$ matrix that defines graph connectivity

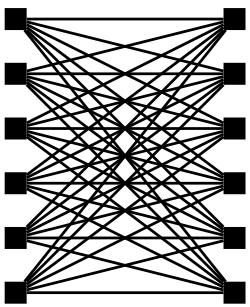
Parameterized Construction of Generalized Product Codes

product codes



staircase codes





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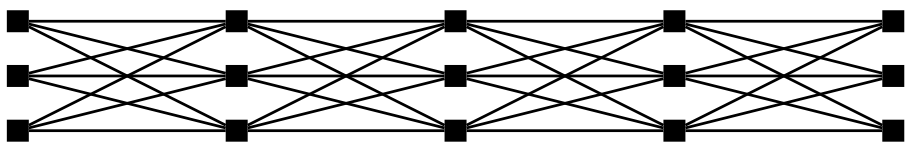
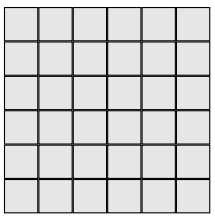


$$\boldsymbol{\eta} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

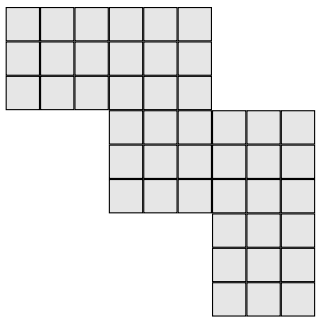
$\boldsymbol{\eta}$: symmetric $L \times L$ matrix that defines graph connectivity

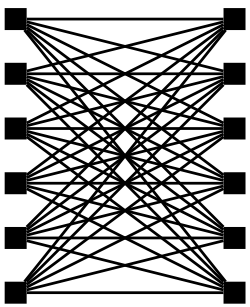
Parameterized Construction of Generalized Product Codes

product codes



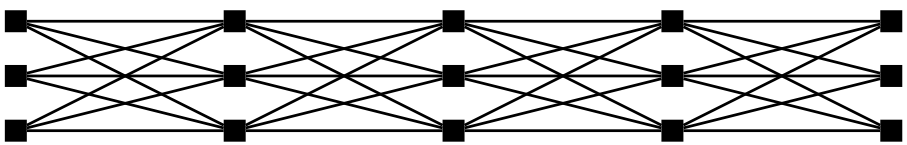
staircase codes





1 $L = 2$

positions: 1 2 3 4 $L = 5$



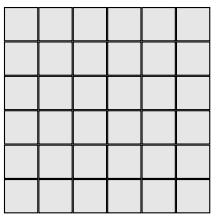
$$\boldsymbol{\eta} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

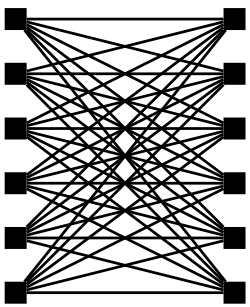
$\boldsymbol{\eta}$: symmetric $L \times L$ matrix that defines graph connectivity

$$\boldsymbol{\eta} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

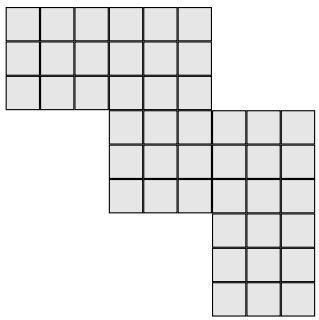
Parameterized Construction of Generalized Product Codes

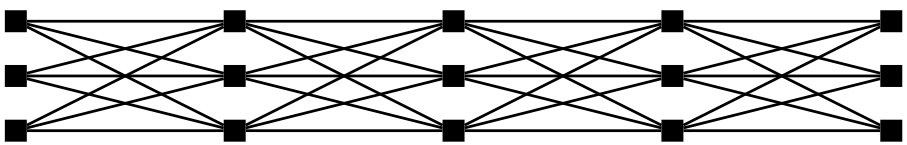
product codes





staircase codes





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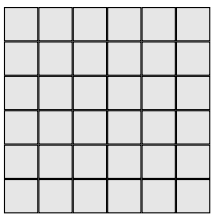

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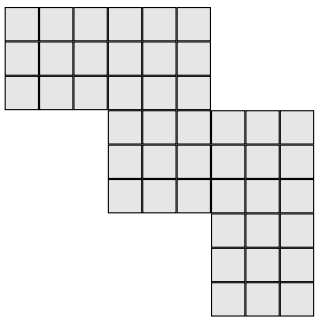
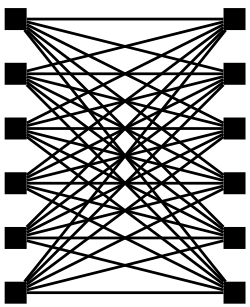
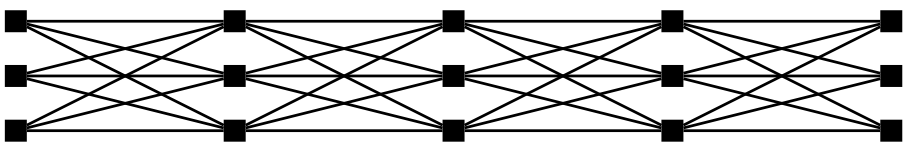




n: "problem size", proportional to the number of constraint nodes



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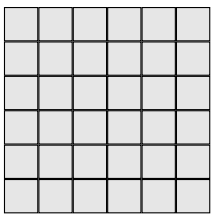

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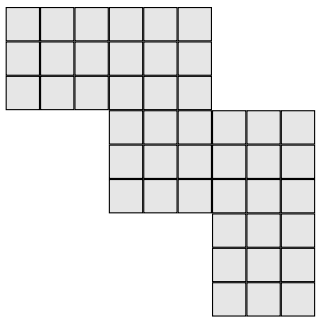

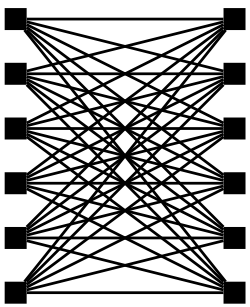

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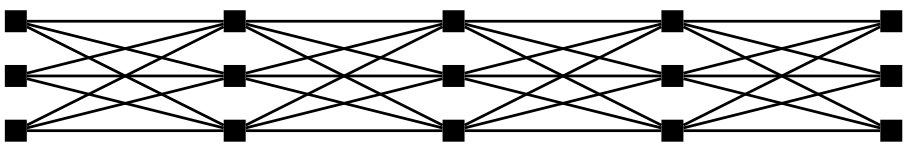
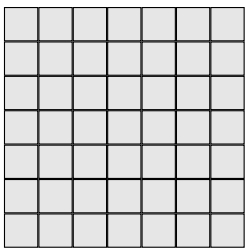


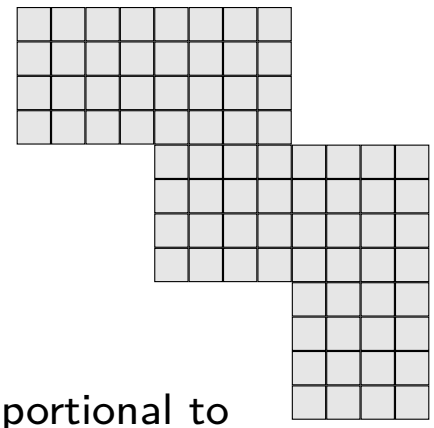
increasing n

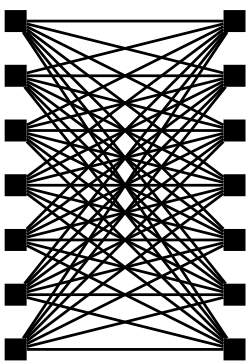
Parameterized Construction of Generalized Product Codes

product codes



staircase codes





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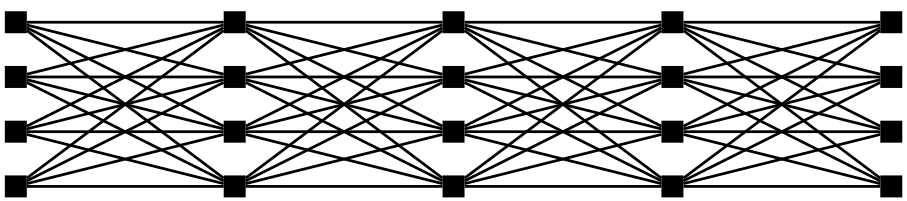

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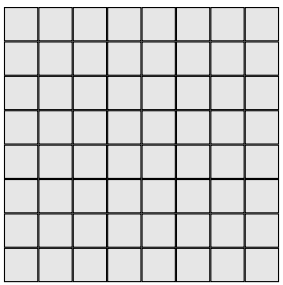

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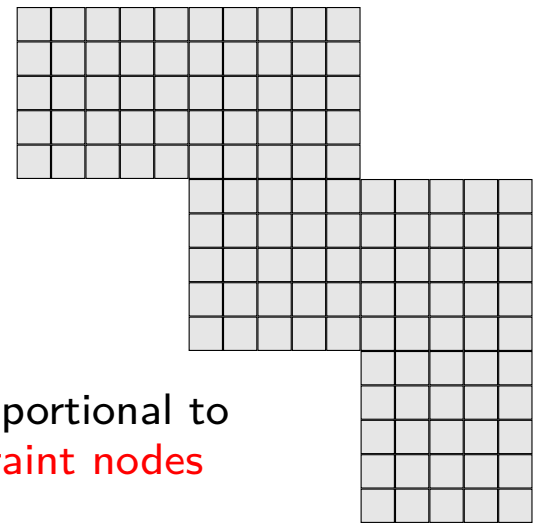

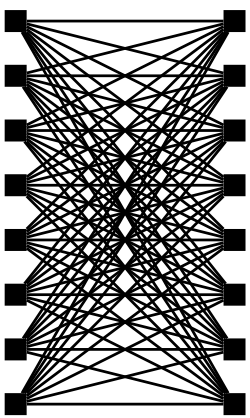

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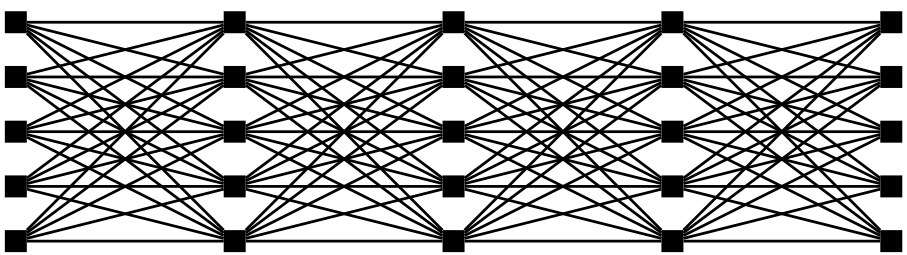
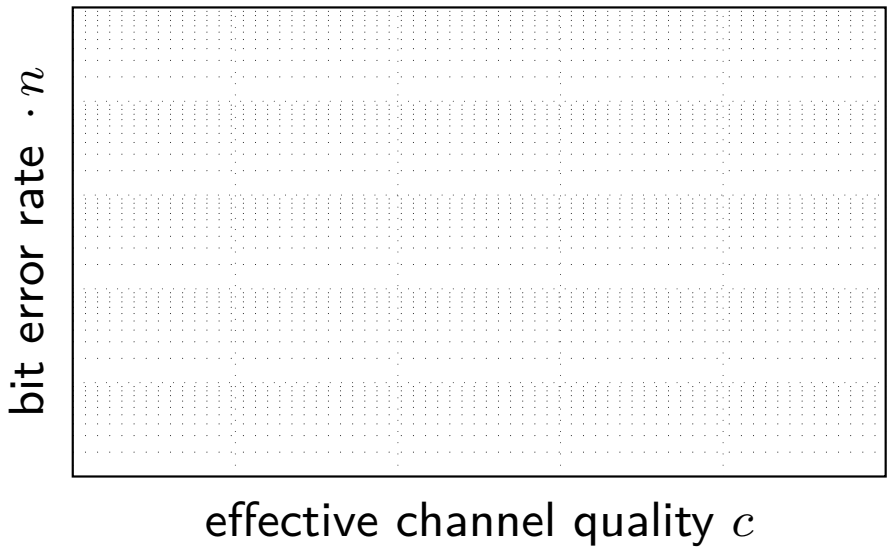


increasing n

Density Evolution

Density Evolution

- Let $p = c/n$ for $c > 0$, where c is the effective channel quality



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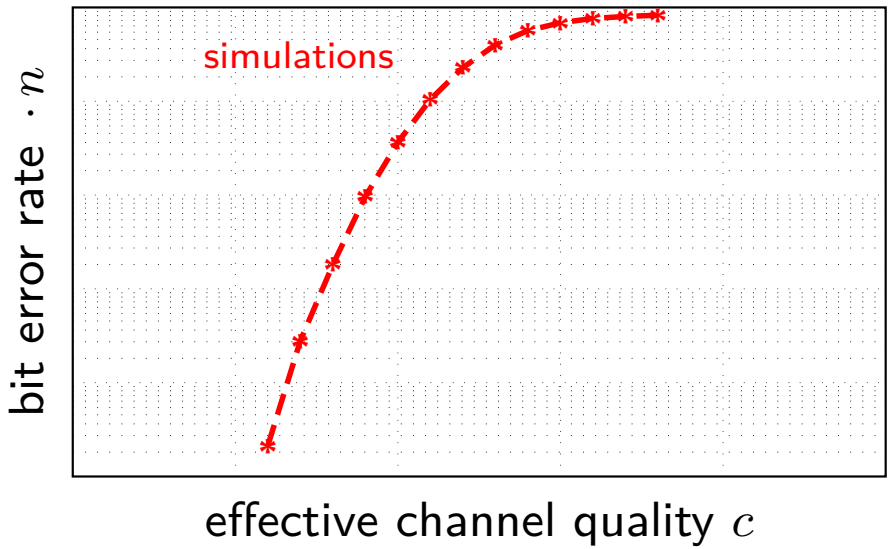

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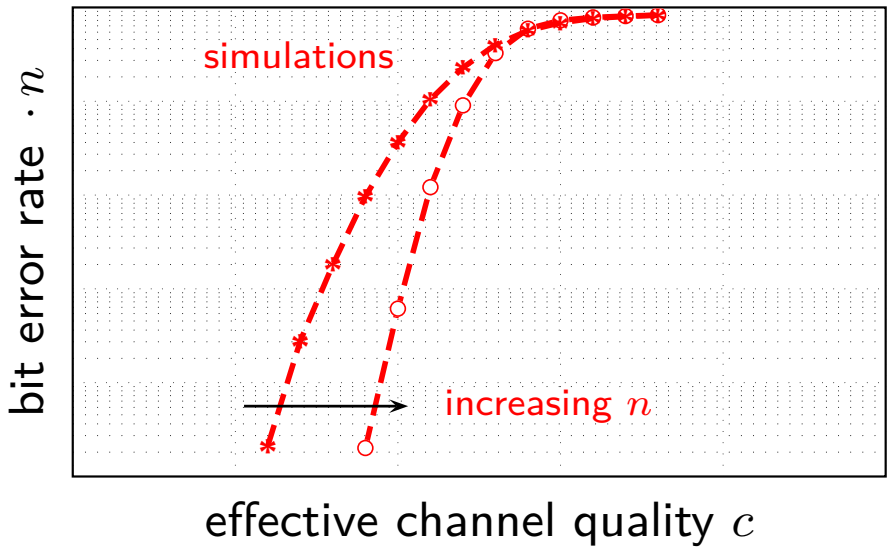

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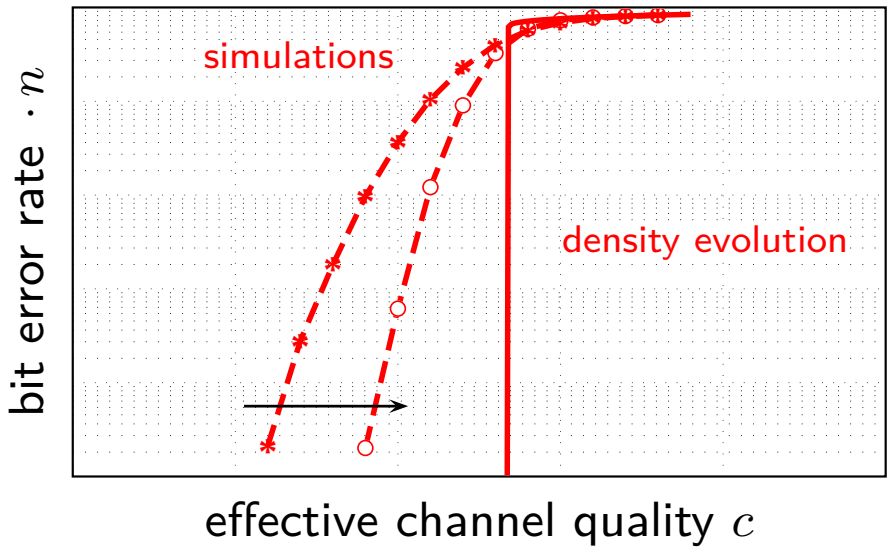

$$\boldsymbol{x}^{(\ell)} = \boldsymbol{\Psi}_{\geq t}(c\boldsymbol{B}\boldsymbol{x}^{(\ell-1)})$$

$\boldsymbol{B} \triangleq \gamma\boldsymbol{\eta}$

initial condition $\boldsymbol{x}^{(0)} = (1, \ldots, 1)$

element-wise application of $\Psi_{\geq t}(x) \triangleq 1 - \sum_{i=0}^{t-1} \frac{x^i}{i!} e^{-x}$

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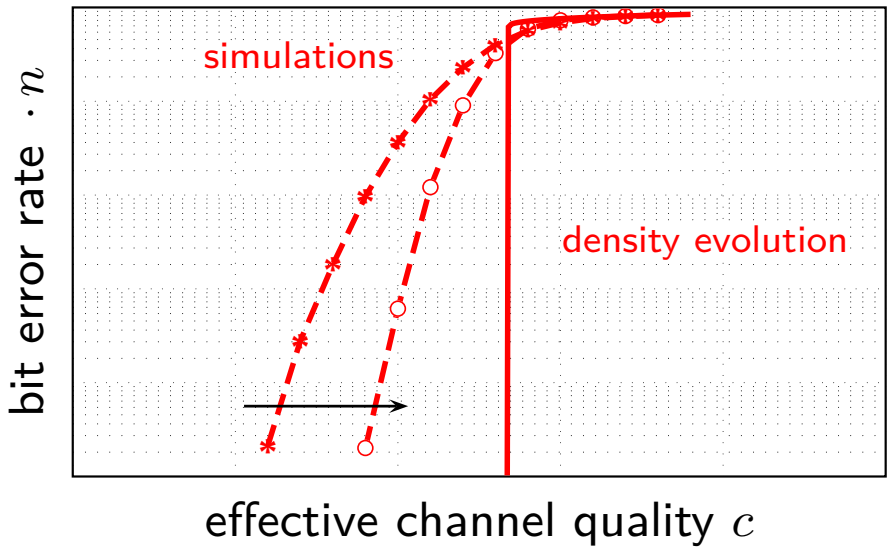
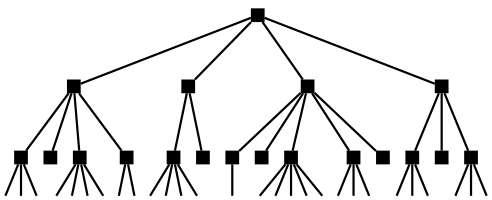


Proof and details in [Häger et al., 2017].
Key: convergence results for sparse random graphs [Bollobás et al., 2007]



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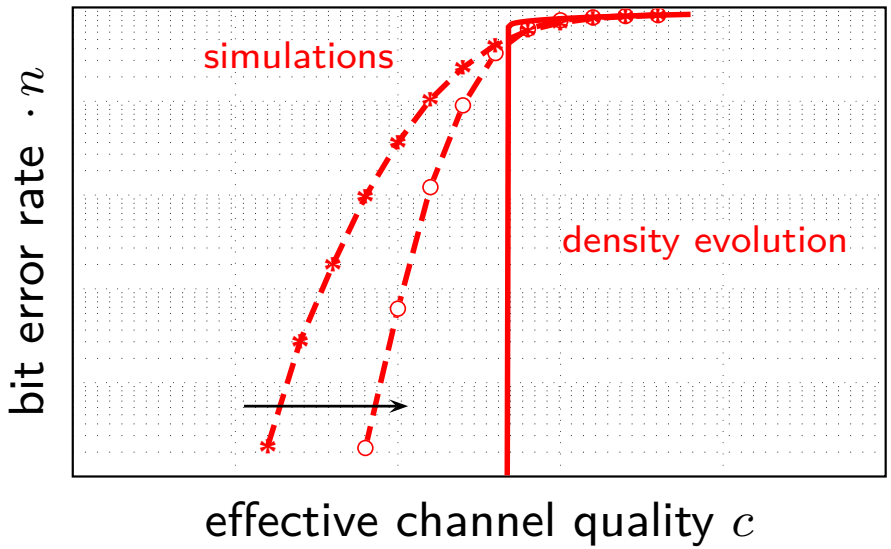

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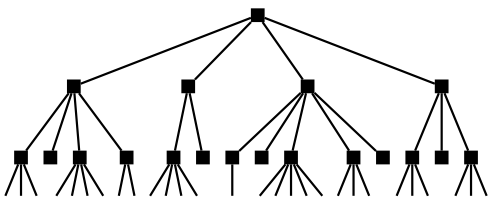

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- Applications: (asymptotic) performance prediction, code comparison via thresholds, efficient parameter optimization, ...

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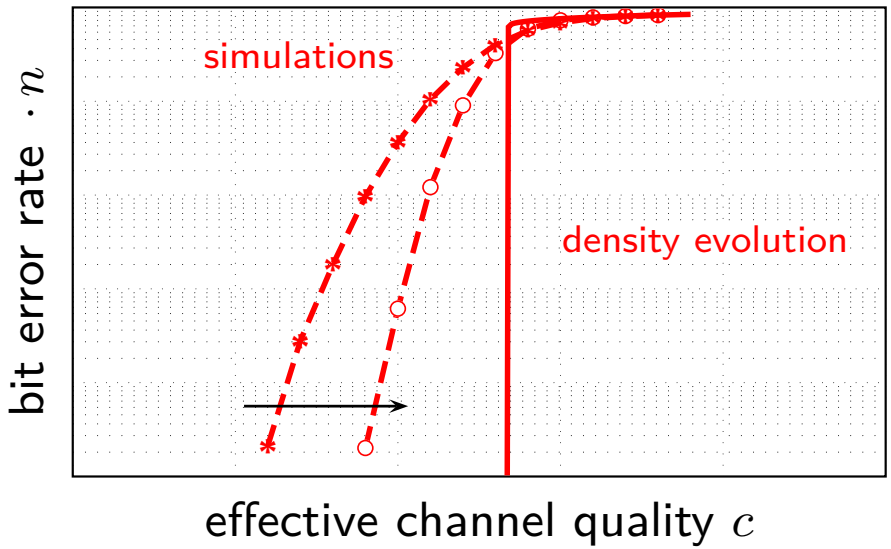

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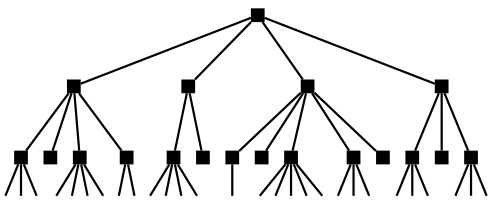

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only one small problem ...

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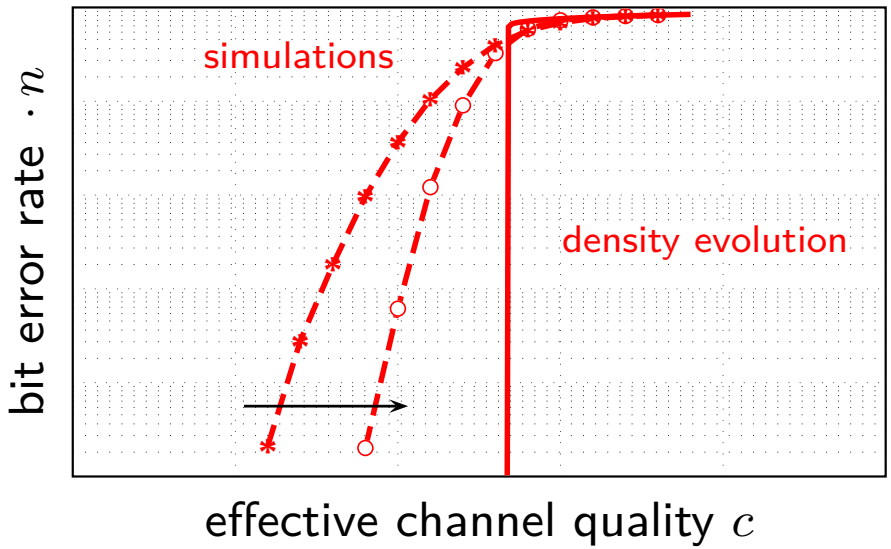

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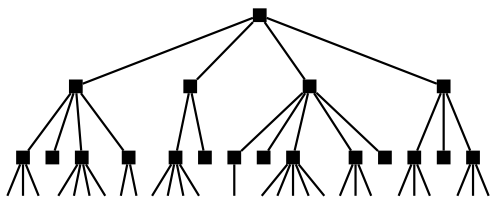

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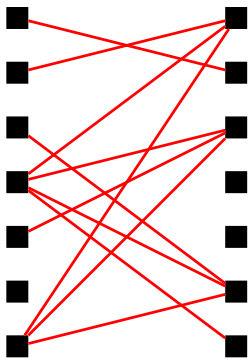
only one small problem ... binary erasure channel is not the target channel

Outline

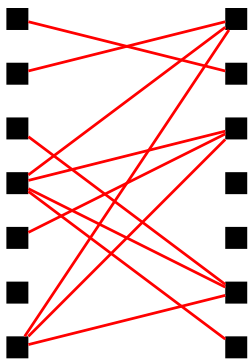
Binary Symmetric Channel vs. Binary Erasure Channel



residual graph

0	?	0	?	0	1	?
?	1	0	1	1	0	1
0	1	0	?	0	?	?
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	?	?	1	1	?
0	1	0	?	0	1	1

Binary Symmetric Channel vs. Binary Erasure Channel



residual graph

0	0	0	0	0	1	1
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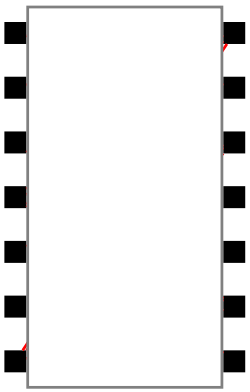

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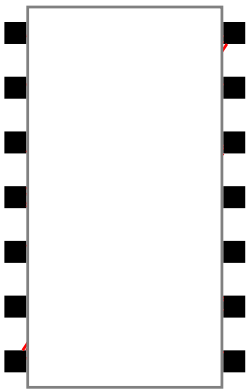

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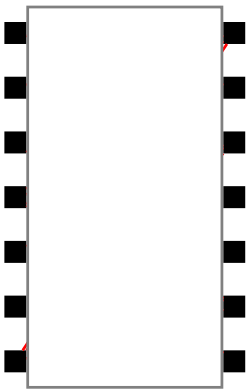
- Each component code corrects $\leq t$ errors

Binary Symmetric Channel vs. Binary Erasure Channel



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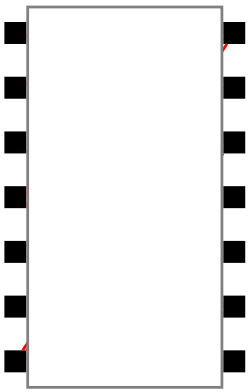

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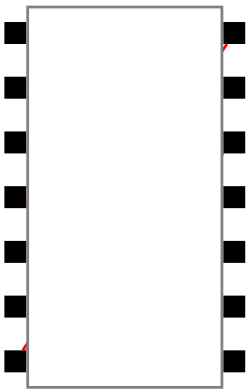

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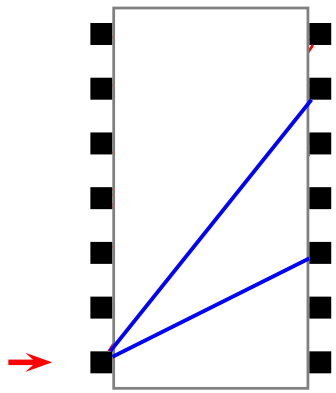

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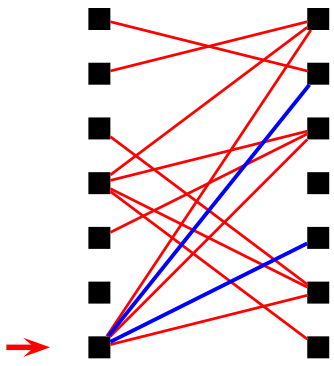

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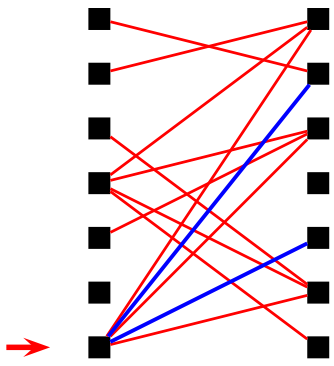
0	0	0	0	0	1	1
1	1	0	1	1	0	0
0	1	0	0	0	0	1
1	1	1	0	1	1	0
0	0	1	0	0	0	0
1	0	1	1	1	1	0
0	1	0	1	0	1	1

- Each component code corrects $\leq t$ errors
- Undetected errors during component decoding $\Longrightarrow$ miscorrections

Binary Symmetric Channel vs. Binary Erasure Channel



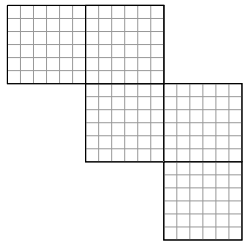
residual graph

0	0	0	0	0	1	1
1	1	0	1	1	0	0
0	1	0	0	0	0	1
1	1	1	0	1	1	0
0	0	1	0	0	0	0
1	0	1	1	1	1	0
0	1	0	1	0	1	1

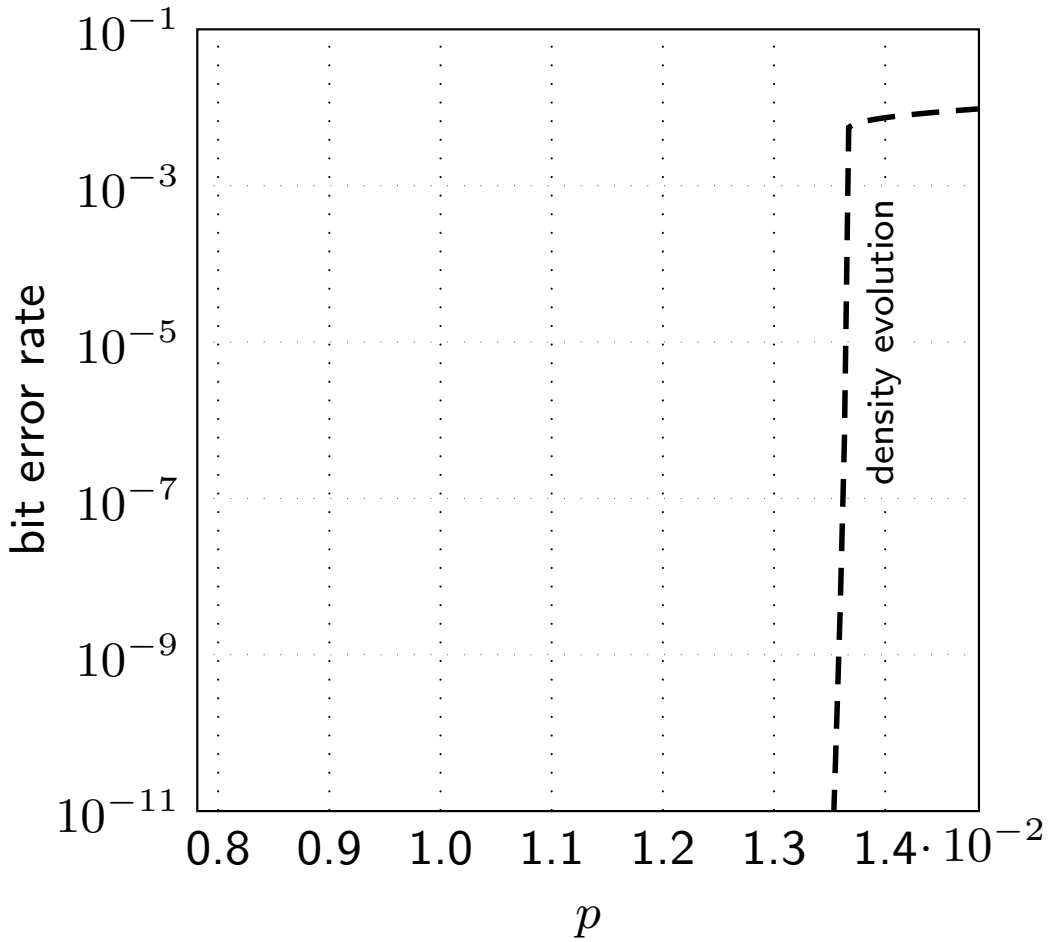
- Each component code corrects $\leq t$ errors
- Undetected errors during component decoding $\Longrightarrow$ miscorrections
- Additional errors during iterative decoding

Performance Loss

Performance Loss



- Staircase code with $n = 256$ and $t = 2$



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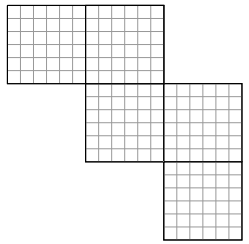

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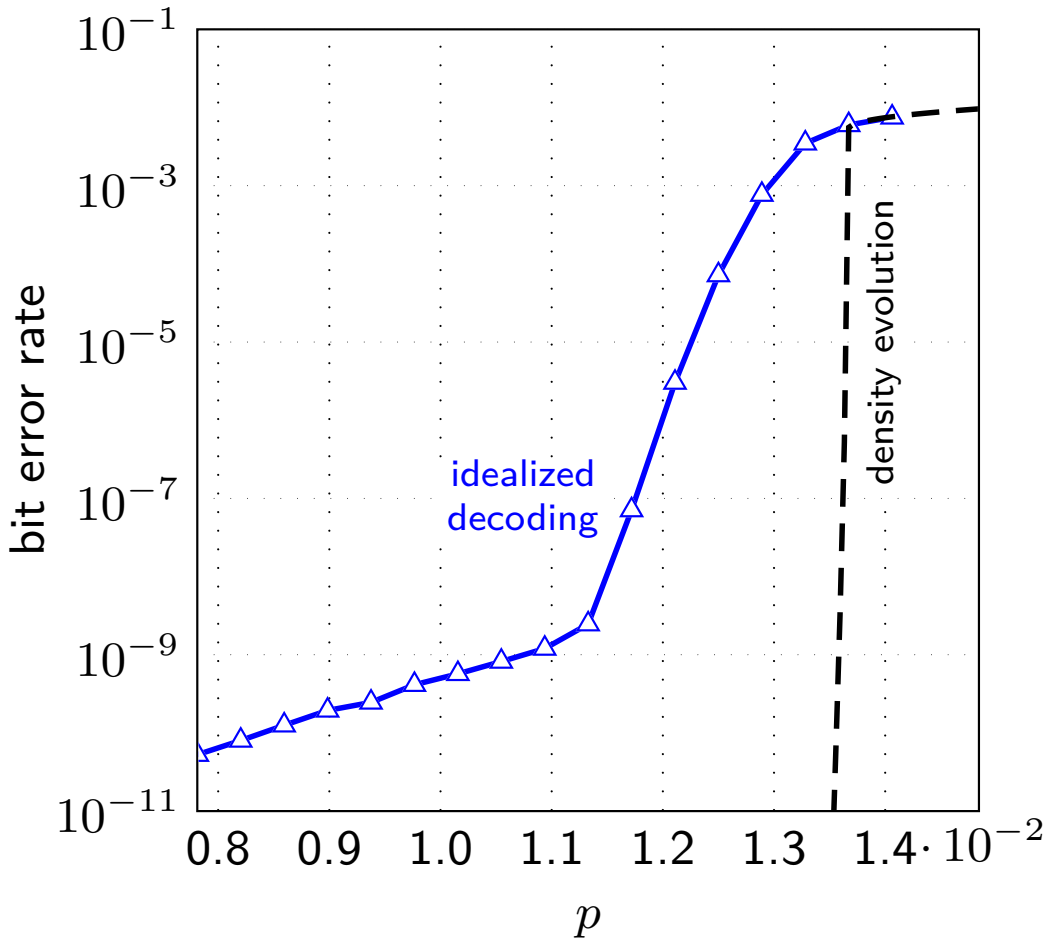
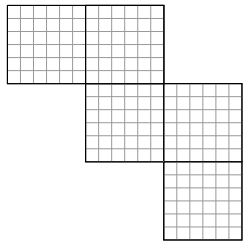
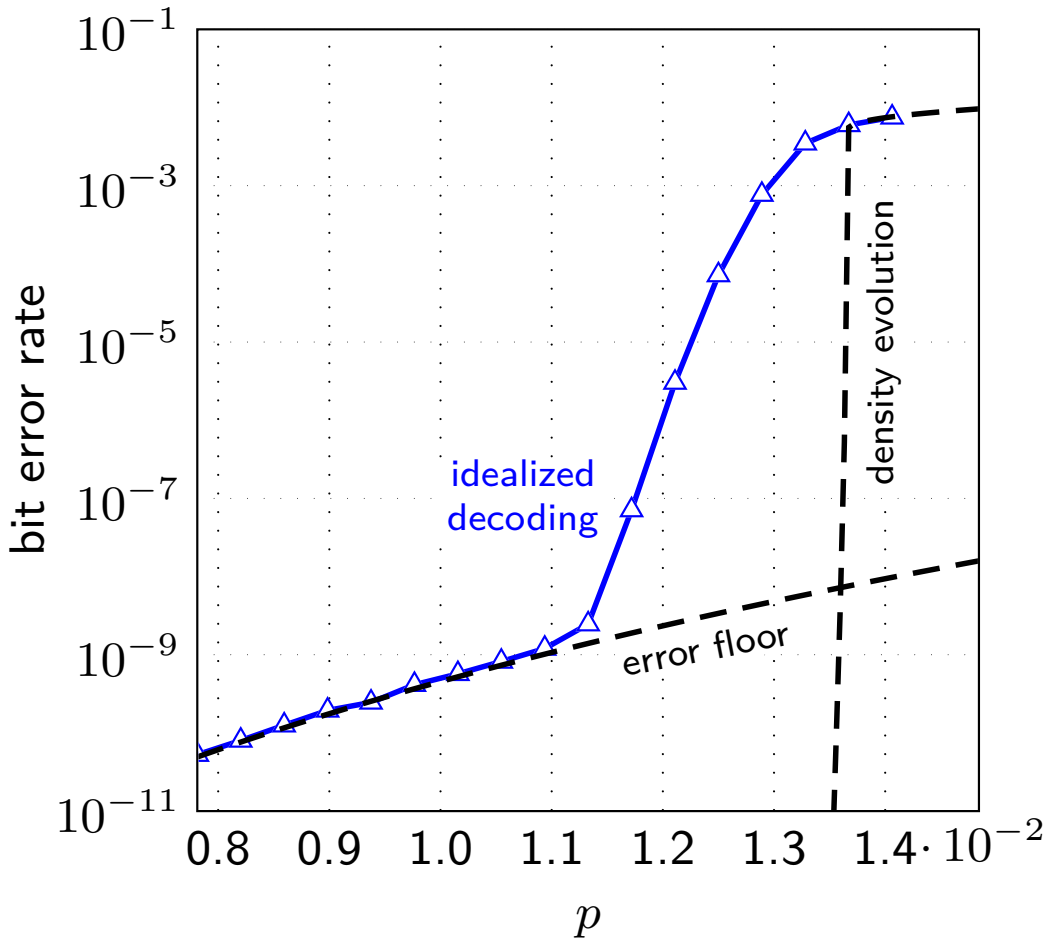


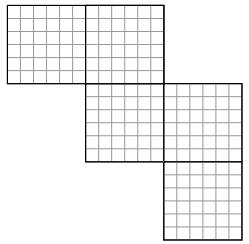
Performance Loss



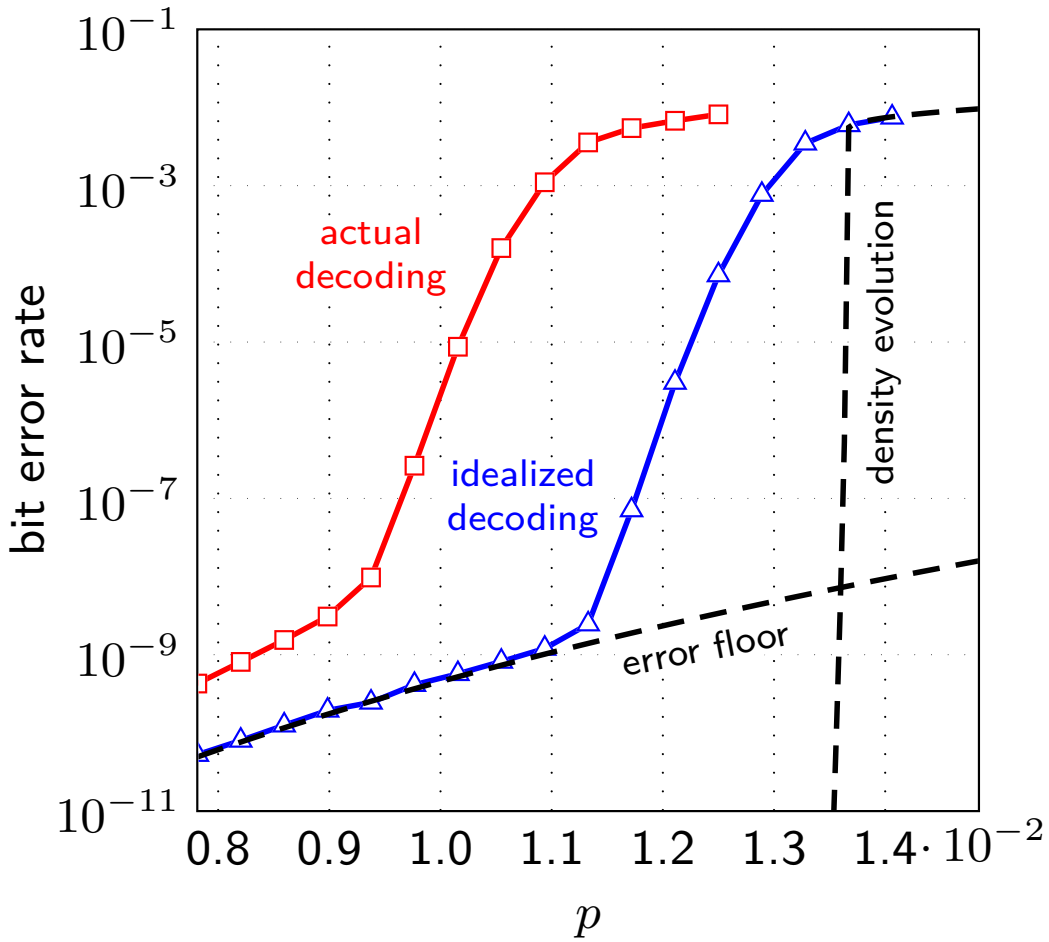
- Staircase code with $n = 256$ and $t = 2$



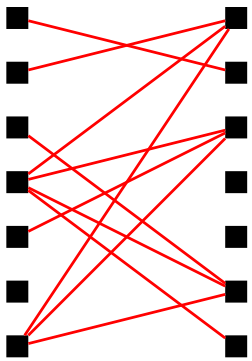
Performance Loss



- Staircase code with $n = 256$ and $t = 2$



Anchor Decoding



residual graph

0	0	0	0	0	1	1
1	1	0	1	1	0	1
0	1	0	0	0	0	1
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	1	1	1	1	0
0	1	0	1	0	1	1

Anchor Decoding

residual graph

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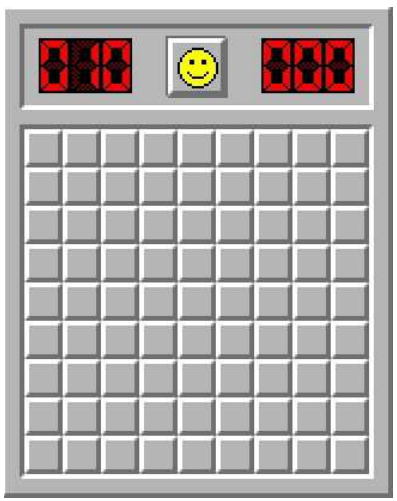
1	1	0	1	1	0	1
0	1	0	0	0	0	1
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	1	1	1	1	0
0	1	0	1	0	1	1

Anchor Decoding

residual graph

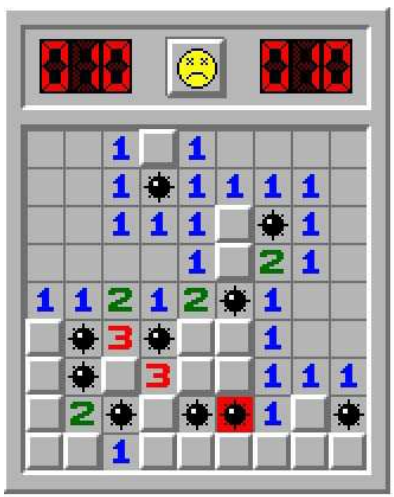
0	0	0	0	0	1	1
1	1	0	1	1	0	1
0	1	0	0	0	0	1
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	1	1	1	1	0
0	1	0	1	0	1	1



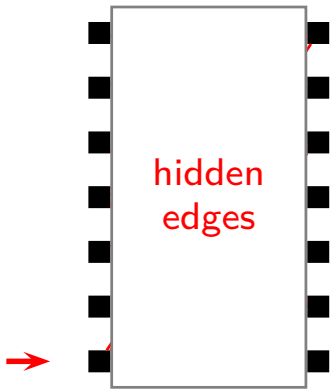
Anchor Decoding

residual graph

0	0	0	0	0	1	1
1	1	0	1	1	0	1
0	1	0	0	0	0	1
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	1	1	1	1	0
0	1	0	1	0	1	1

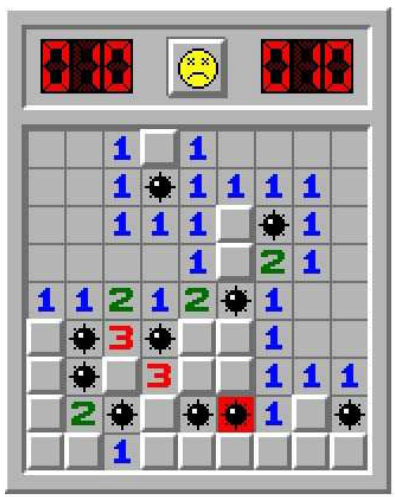


Anchor Decoding

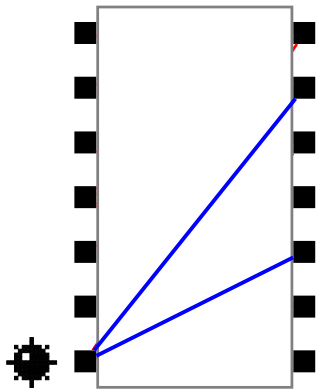


residual graph

0	0	0	0	0	1	1
1	1	0	1	1	0	1
0	1	0	0	0	0	1
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	1	1	1	1	0
0	1	0	1	0	1	1

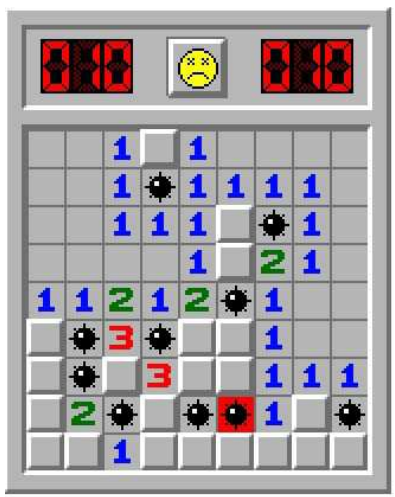


Anchor Decoding

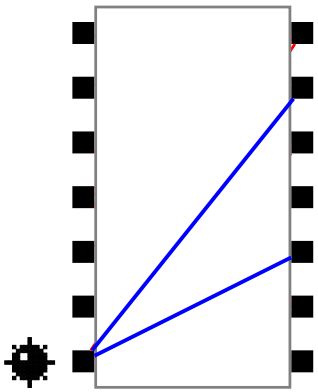


residual graph

0	0	0	0	0	1	1
1	1	0	1	1	0	0
0	1	0	0	0	0	1
1	1	1	0	1	1	0
0	0	1	0	0	0	0
1	0	1	1	1	1	0
0	1	0	1	0	1	1



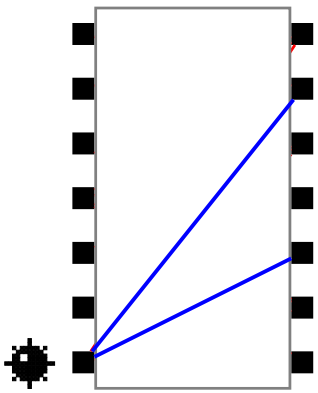
Anchor Decoding



residual graph

0	0	0	0	0	1	1
1	1	0	1	1	0	0
0	1	0	0	0	0	1
1	1	1	0	1	1	0
0	0	1	0	0	0	0
1	0	1	1	1	1	0
0	1	0	1	0	1	1

Anchor Decoding

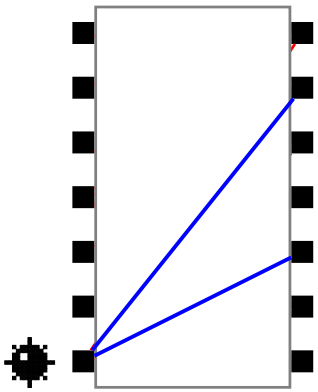


residual graph

0	0	0	0	0	1	1
1	1	0	1	1	0	0
0	1	0	0	0	0	1
1	1	1	0	1	1	0
0	0	1	0	0	0	0
1	0	1	1	1	1	0
0	1	0	1	0	1	1

- Miscorrections lead to inconsistencies/conflicts: two component codewords may disagree on the value of a bit

Anchor Decoding



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0	1	0	0	0	0	1
1	1	1	0	1	1	0
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1	0	1	1	1	1	0
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- Idea: make correctly decoded codewords anchors and trust their decisions (requires status information for each component codeword)

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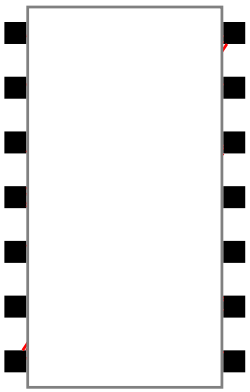

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0	1	0	0	0	0	1
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	1	1	1	1	0
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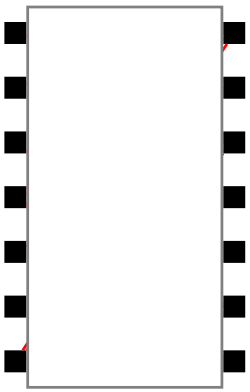

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0	1	0	0	0	0	1
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	1	1	1	1	0
0	1	0	1	0	1	1

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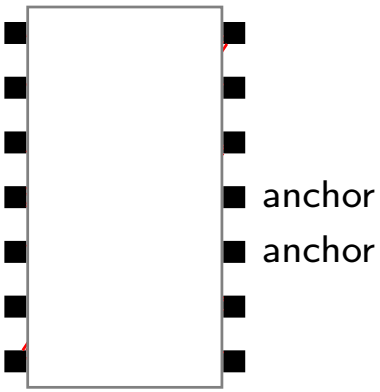

0	0	0	0	0	1	1
1	1	0	1	1	0	1
0	1	0	0	0	0	1
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	1	1	1	1	0
0	1	0	1	0	1	1

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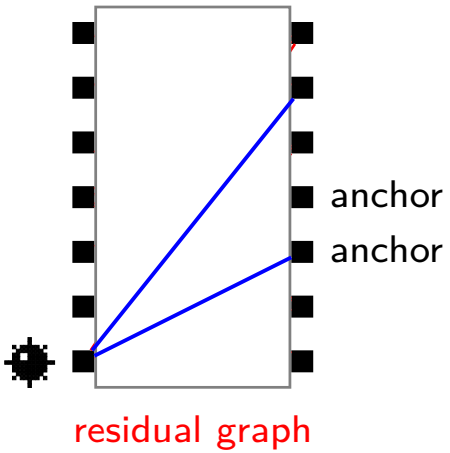
Anchor Decoding



0	0	0	0	0	1	1
1	1	0	1	1	0	0
0	1	0	0	0	0	1
1	1	1	0	1	1	0
0	0	1	0	0	0	0
1	0	1	1	1	1	0
0	1	0	1	0	1	1

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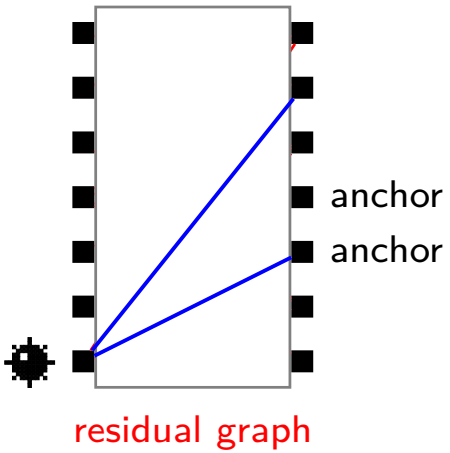

0	0	0	0	0	1	1
1	1	0	1	1	0	0
0	1	0	0	0	0	1
1	1	1	0	1	1	0
0	0	1	0	0	0	0
1	0	1	1	1	1	0
0	1	0	1	0	1	1

conflict with anchor
$\Longrightarrow$ reject bit flips

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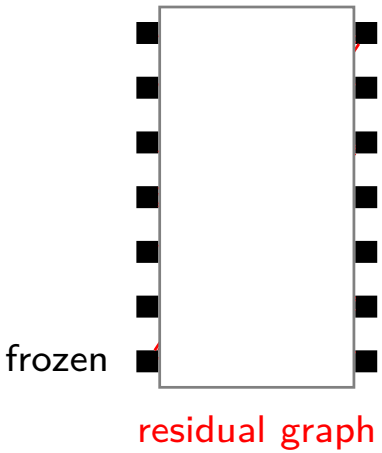

0	0	0	0	0	1	1
1	1	0	1	1	0	1
0	1	0	0	0	0	1
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	1	1	1	1	0
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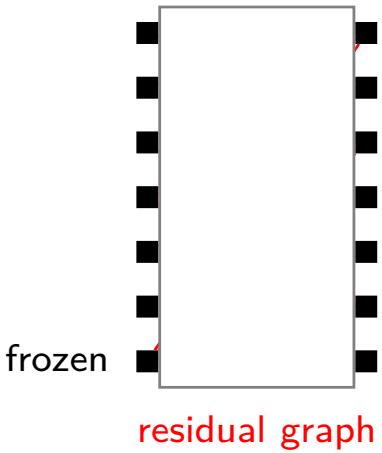
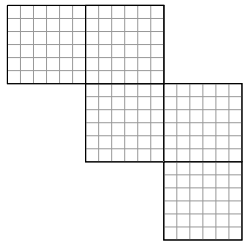


0	0	0	0	0	1	1
1	1	0	1	1	0	1
0	1	0	0	0	0	1
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	1	1	1	1	0
0	1	0	1	0	1	1

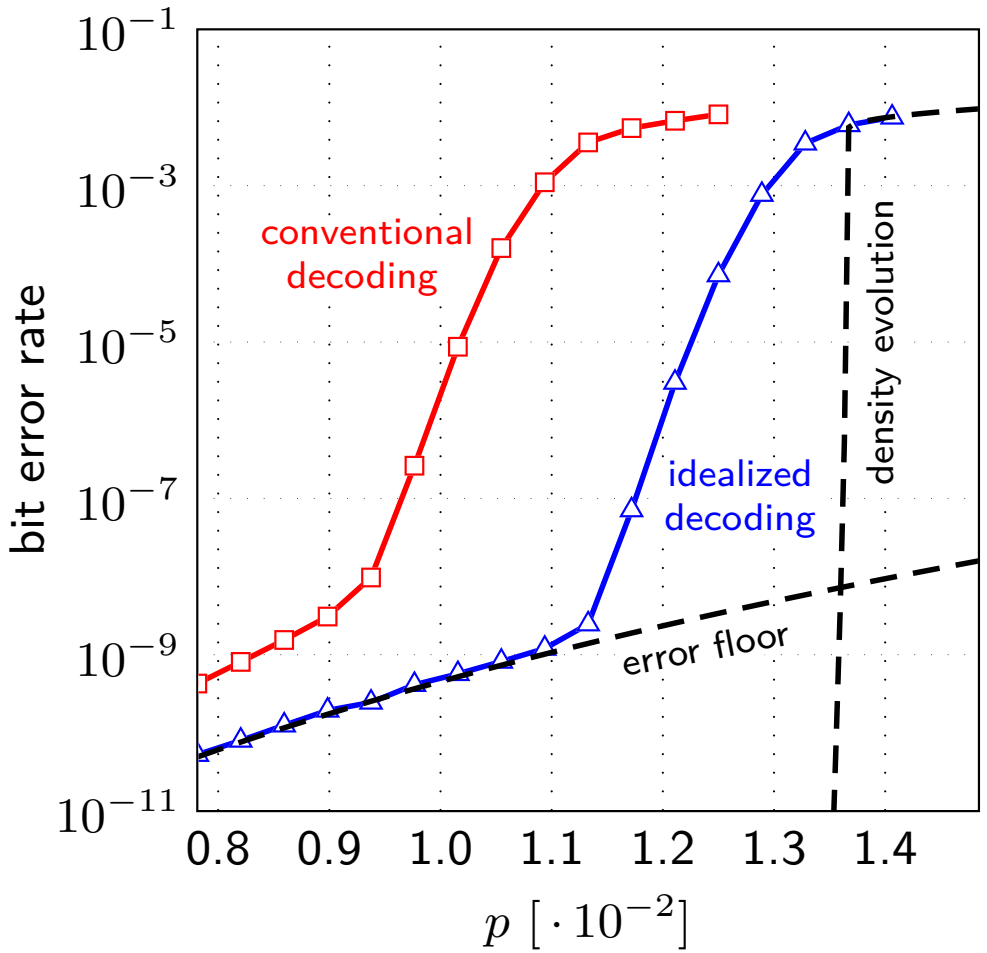
conflict with anchor
$\Longrightarrow$ reject bit flips

- Miscorrections lead to inconsistencies/conflicts: two component codewords may disagree on the value of a bit
- Idea: make correctly decoded codewords anchors and trust their decisions (requires status information for each component codeword)
- If any anchor has too many conflicts, backtrack its bit flips

Simulation Results



- Staircase code with $n = 256$ and $t = 2$



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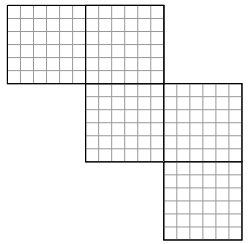

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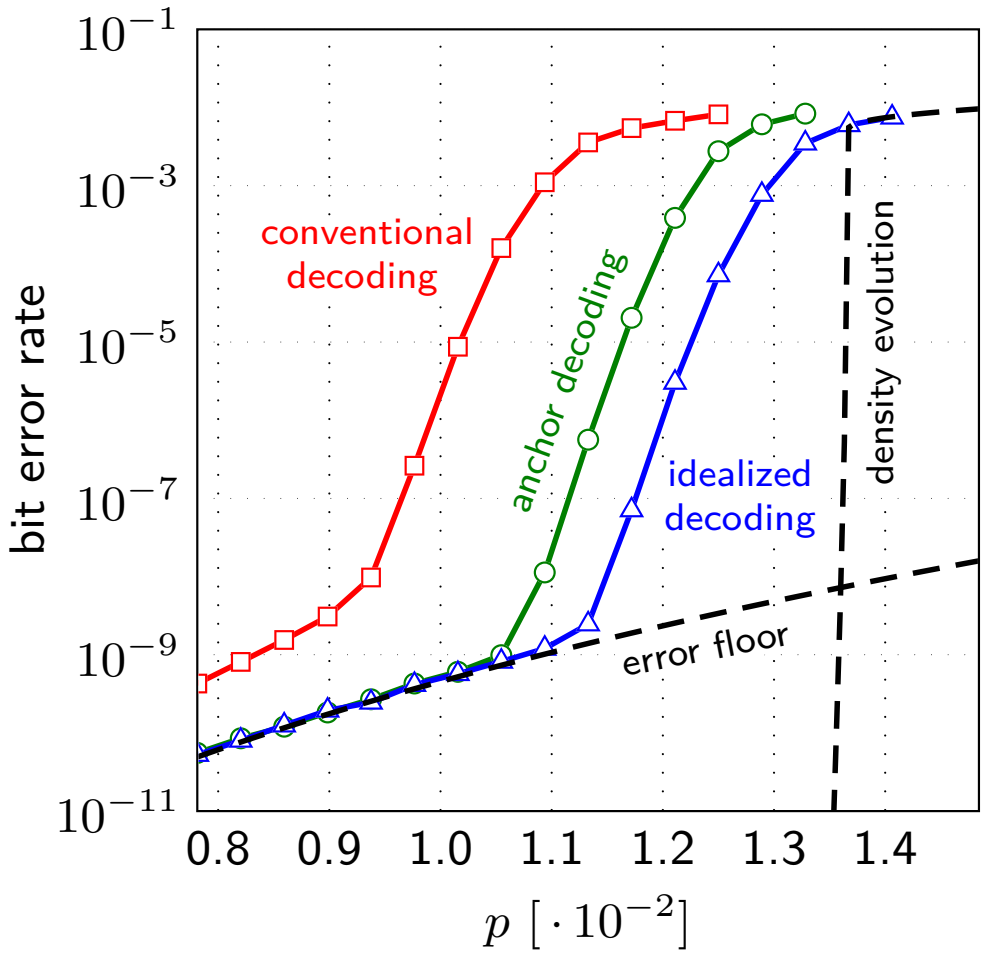
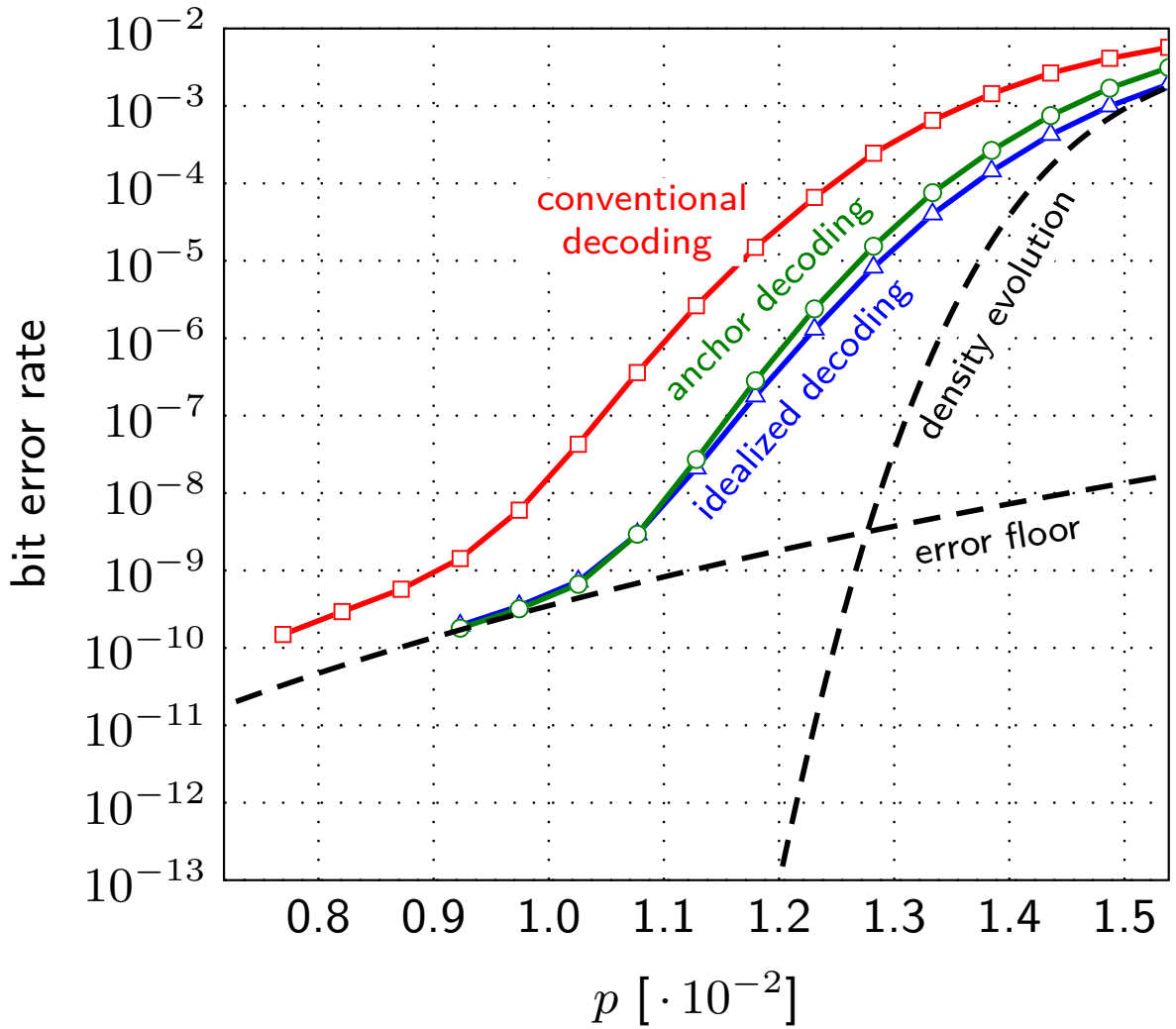


Simulation Results (cont.)

- Product code with $n = 195$ and $t = 2$, see [Condo et al., 2018]



Simulation Results (cont.)

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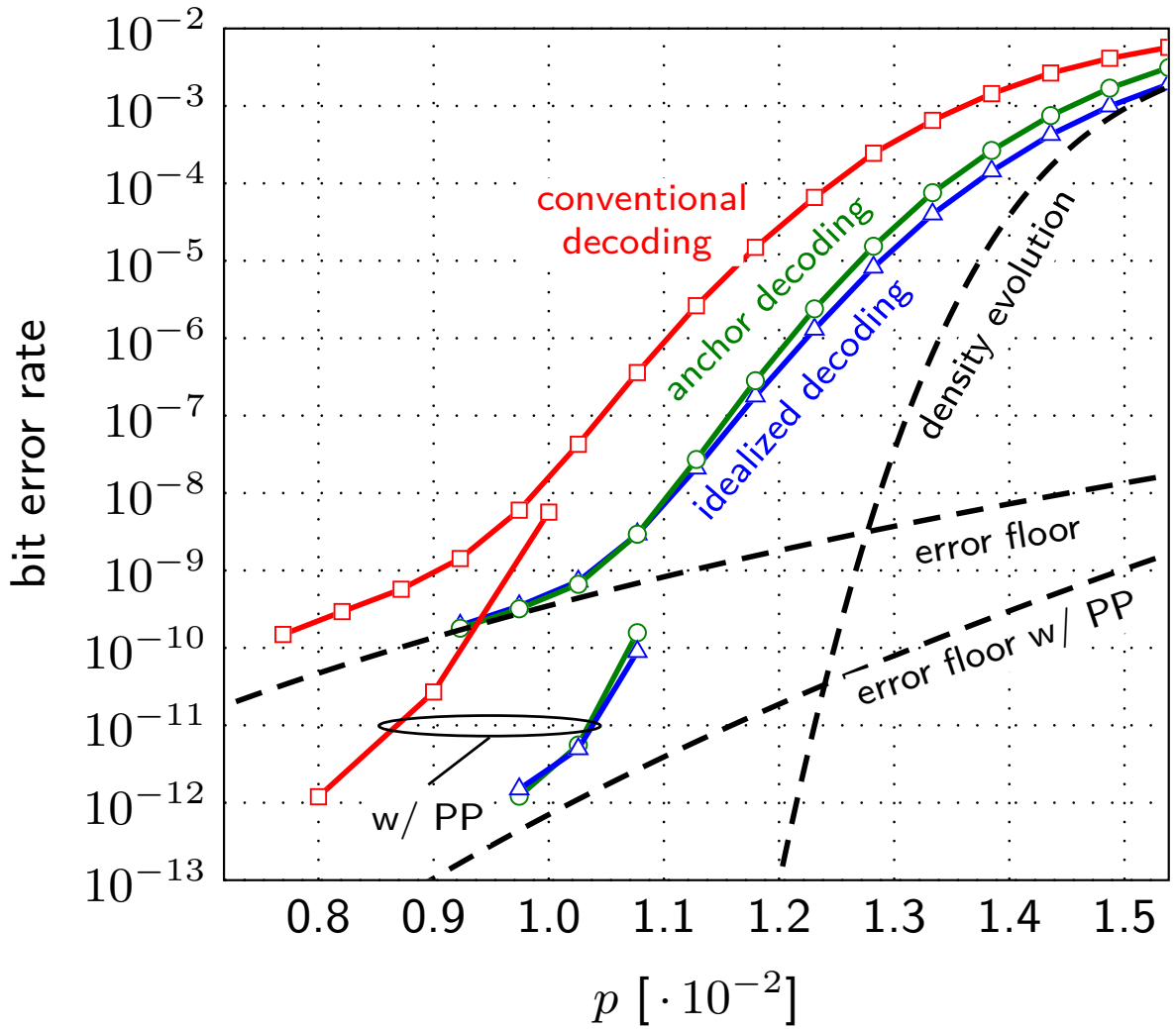

post-processing (PP):
[Jian et al., 2014]
[Mittelholzer et al., 2016]
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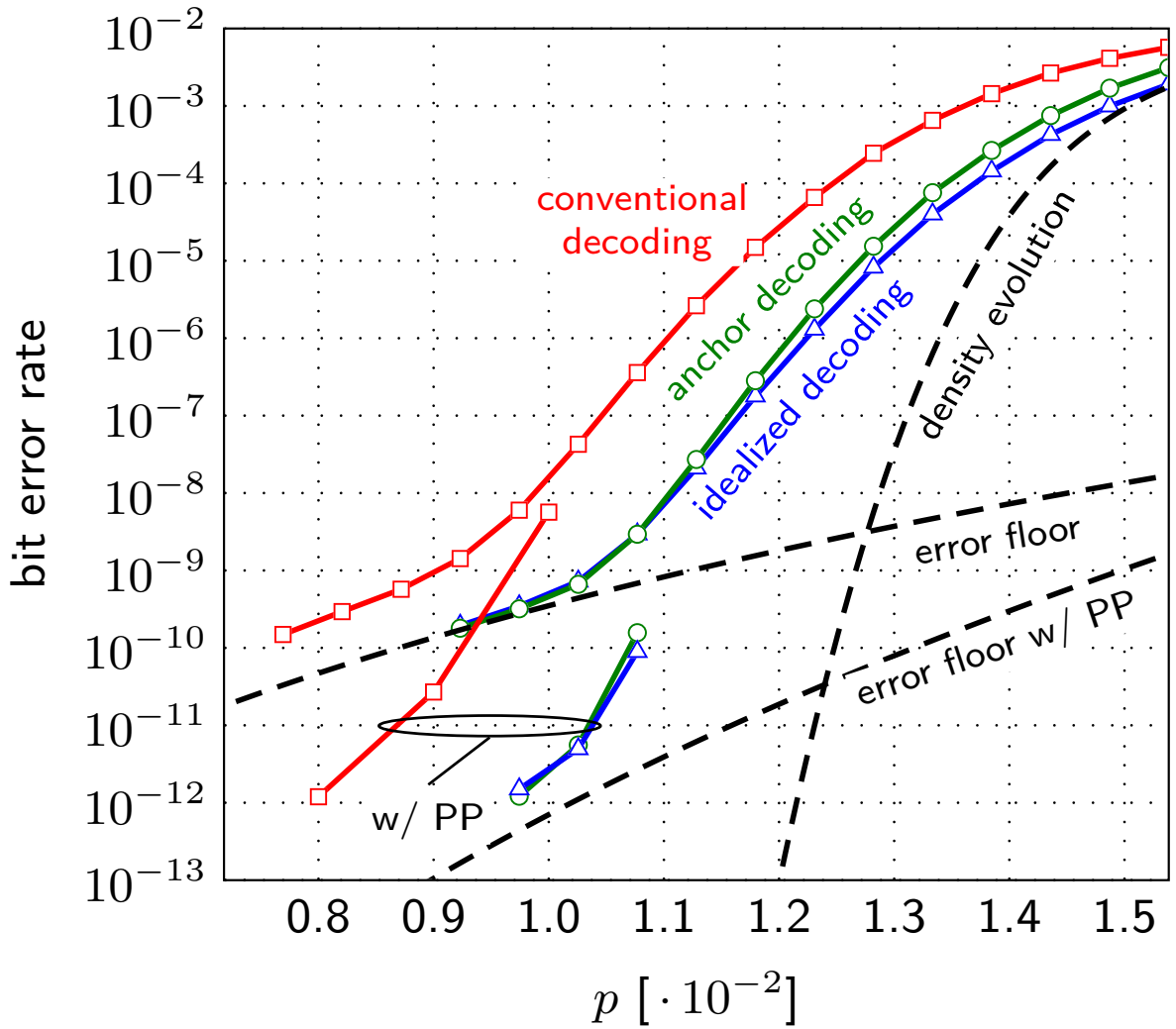
Simulation Results (cont.)

- Product code with $n = 195$ and $t = 2$, see [Condo et al., 2018]



post-processing (PP):
[Jian et al., 2014]
[Mittelholzer et al., 2016]
[Holzbaur et al., 2017]

Future work: PP for staircase codes, complexity impact on product decoder architecture, ...

Conclusions

- Density evolution can be applied for deterministic generalized product codes over the binary erasure channel.
- In practice, miscorrection-free performance over the binary symmetric channel can be approached with anchor decoding.

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Thank you!

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