

Deep Learning of the Nonlinear Schrödinger Equation in Fiber-Optic Communications

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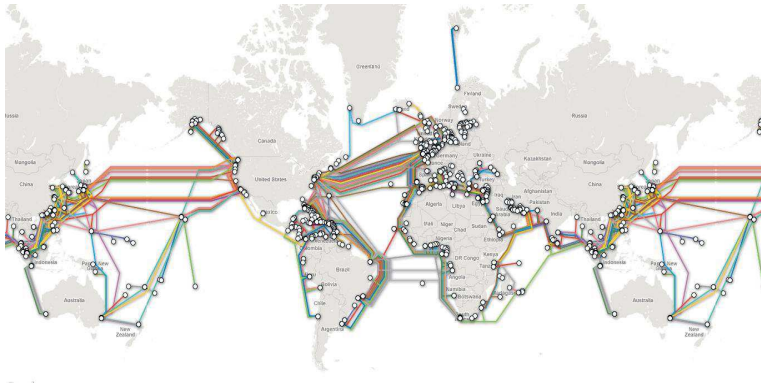
ISIT 2018, Vail, June 21, 2018



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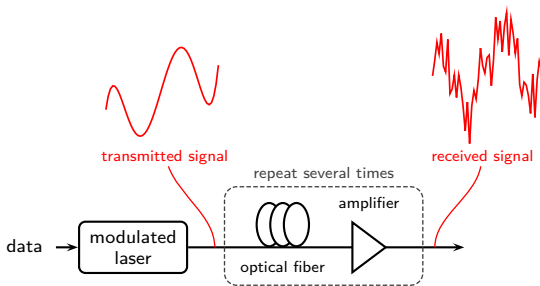


Fiber-Optic Communications

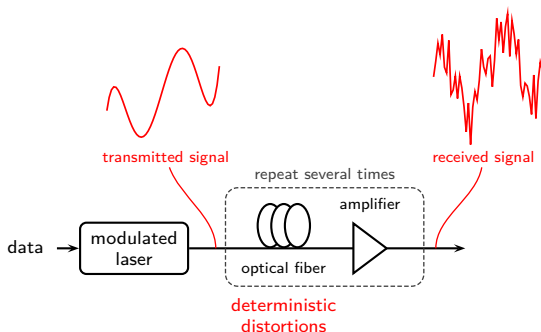


Fiber-optic communication systems enable **high-speed data traffic (100 Gbit/s per channel or higher)** over very long distances.

Fiber-Optic Communications

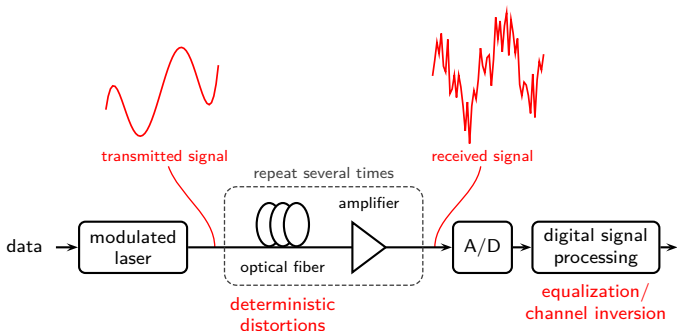


Fiber-Optic Communications



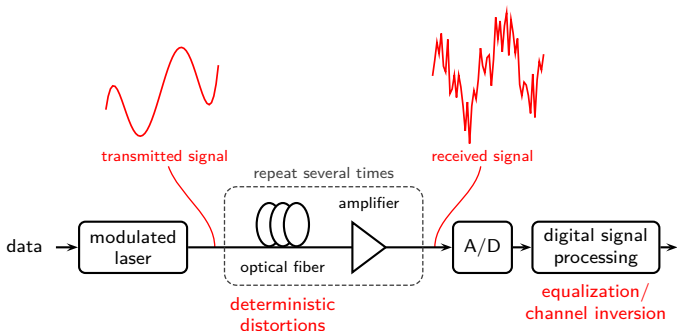
- **Dispersion:** different wavelengths travel at different speeds (linear)
- **Kerr effect:** refractive index changes with signal intensity (nonlinear)

Fiber-Optic Communications



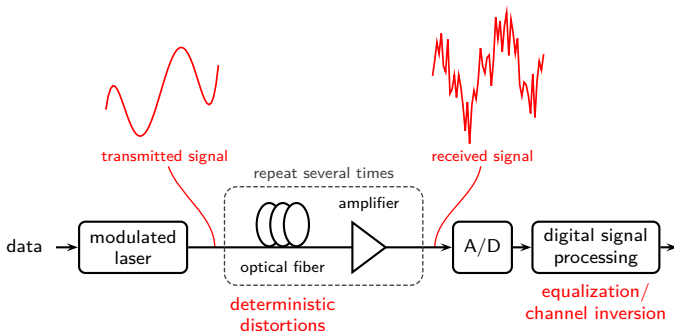
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Fiber-Optic Communications



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- **Key challenge:** complexity constraints due to very high data rates

Fiber-Optic Communications



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This talk

Machine learning for low-complexity **real-time** channel inversion

Outline

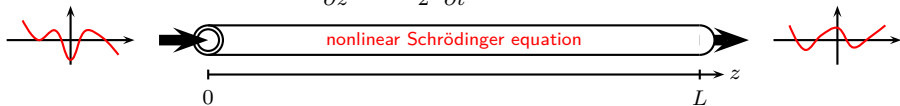
1. Channel Modeling and the Nonlinear Schrödinger Equation
2. Connection to Deep Learning
3. Joint Filter Optimization and Pruning
4. Results
5. Conclusions

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Channel Modeling and Digital Backpropagation

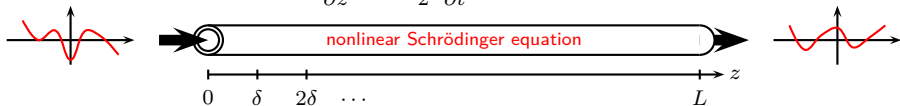
$$\frac{\partial u}{\partial z} = -j\frac{\beta_2}{2}\frac{\partial^2 u}{\partial t^2} + j\gamma u|u|^2$$



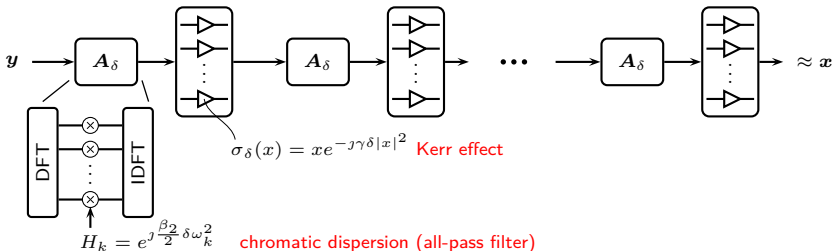
- Invert a partial differential equation **in real time** ([Paré et al., 1996], [Essiambre and Winzer, 2005], [Roberts et al., 2006], [Li et al., 2008], [Ip and Kahn, 2008])

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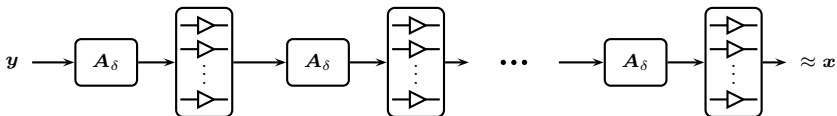
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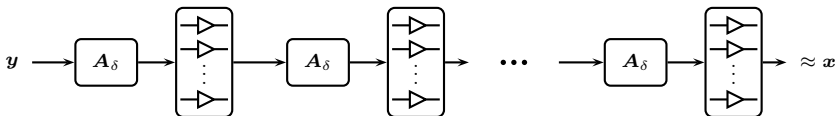
- Invert a partial differential equation **in real time** ([Paré et al., 1996], [Essiambre and Winzer, 2005], [Roberts et al., 2006], [Li et al., 2008], [Ip and Kahn, 2008])
- **Split-step Fourier method** with M steps ($\delta = L/M$):



Real-Time Digital Backpropagation

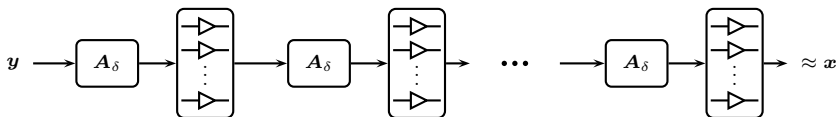


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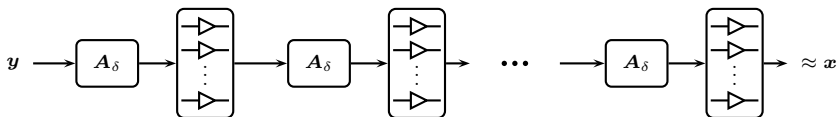
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Real-Time Digital Backpropagation



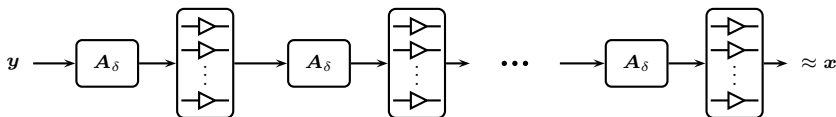
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- Complexity increases with the number of steps $M \implies$ **reduce M as much as possible** (see, e.g., [Du and Lowery, 2010], [Rafique et al., 2011], [Li et al., 2011], [Yan et al., 2011], [Napoli et al., 2014], [Secondini et al., 2016], . . .)

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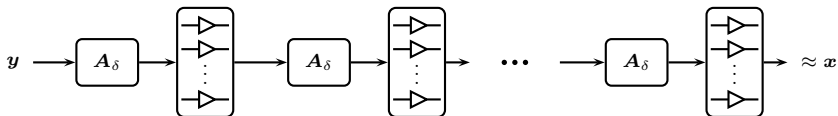
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- Intuitive, but ...

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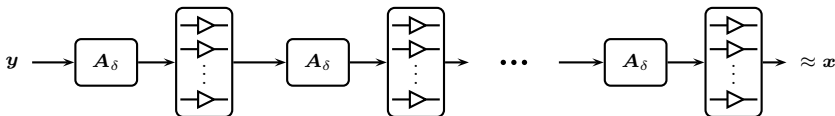
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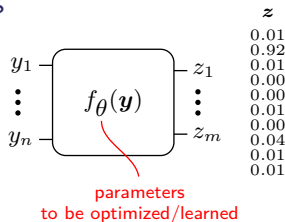
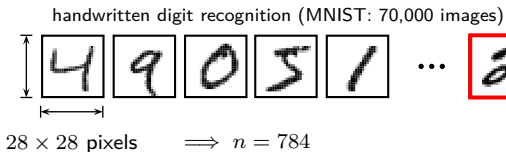
Main contribution

- **Joint optimization** of all linear steps leads to **efficient channel inversion**
- **Power consumption comparable to** published results for **linear equalization**

Outline

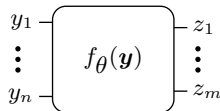
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Supervised Learning

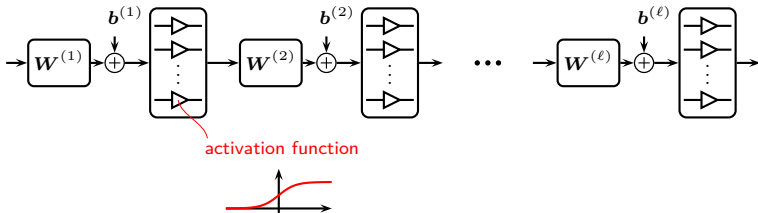


Supervised Learning

handwritten digit recognition (MNIST: 70,000 images)

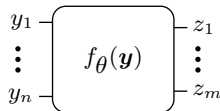

 z
 0.01
 0.92
 0.01
 0.00
 0.00
 0.01
 0.00
 0.04
 0.01
 0.01

How to choose $f_\theta(\mathbf{y})$? **Deep feed-forward neural networks**



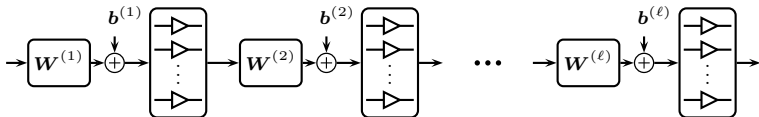
Supervised Learning

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z	x
0.01	0
0.92	1
0.01	0
0.00	0
0.00	0
0.01	0
0.00	0
0.04	0
0.01	0
0.01	0

How to choose $f_\theta(\mathbf{y})$? **Deep feed-forward neural networks**

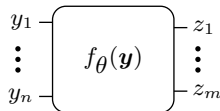


How to optimize $\theta = \{\mathbf{W}^{(1)}, \dots, \mathbf{W}^{(\ell)}, \mathbf{b}^{(1)}, \dots, \mathbf{b}^{(\ell)}\}$? **Deep learning**

$$\min_{\theta} \sum_{i=1}^N \text{Loss}(f_{\theta}(\mathbf{y}^{(i)}), \mathbf{x}^{(i)}) \triangleq g(\theta) \quad \text{using} \quad \theta_{k+1} = \theta_k - \lambda \nabla_{\theta} g(\theta_k) \quad (1)$$

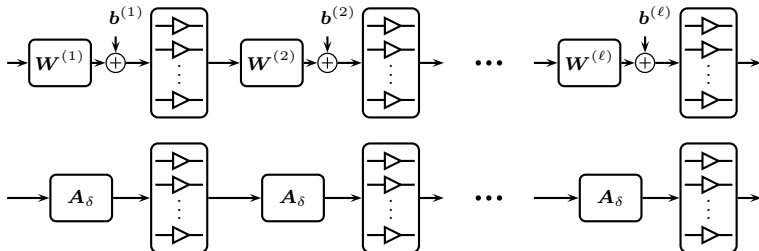
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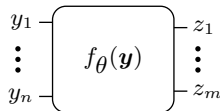
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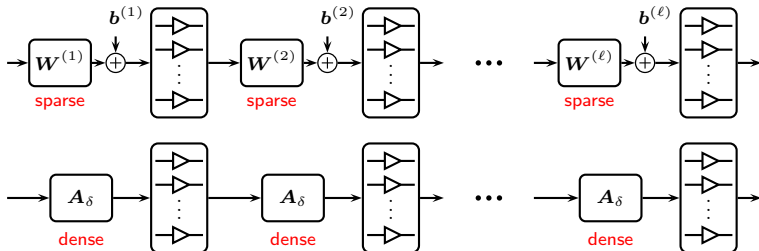
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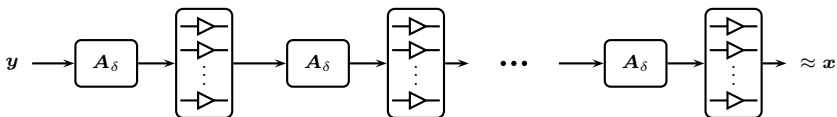


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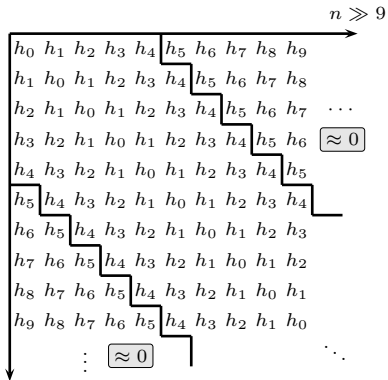
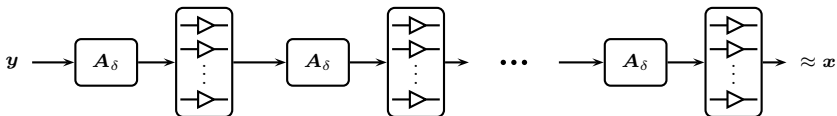
How to choose $f_\theta(\mathbf{y})$? **Deep feed-forward neural networks**



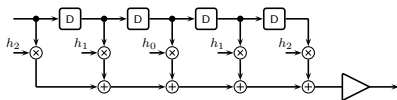
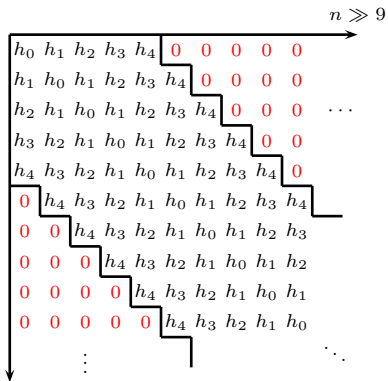
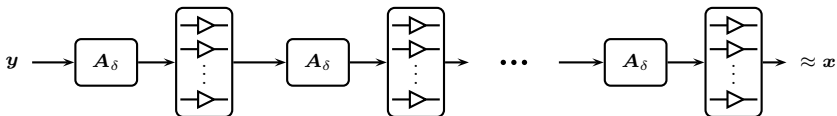
Time-Domain Implementation and Truncation



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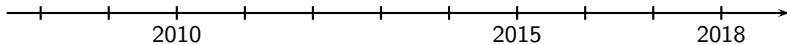


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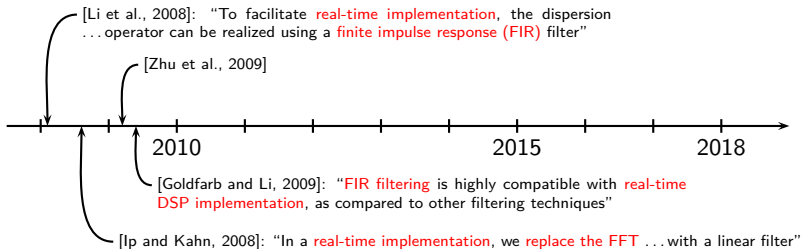


finite impulse response (FIR) filter

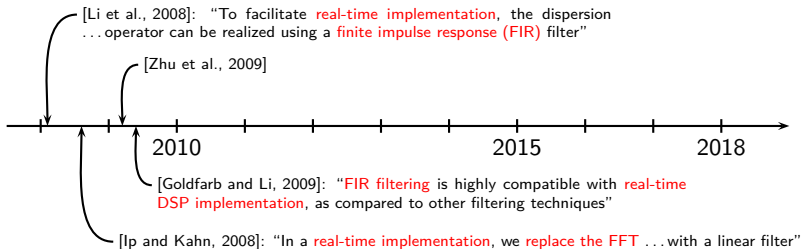
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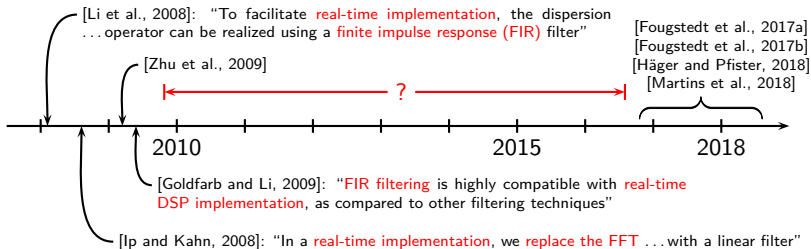


Nontrivial to achieve a good performance–complexity tradeoff!

Example for $R_{\text{symp}} = 10.7$ Gbaud, $L = 2000$ km [Ip and Kahn, 2008]

- $\gg 1000$ total filter taps required for good performance

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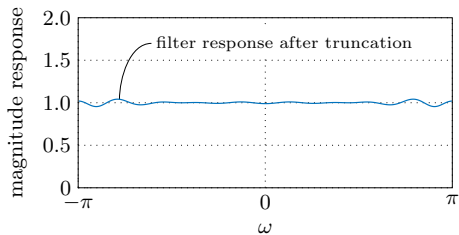


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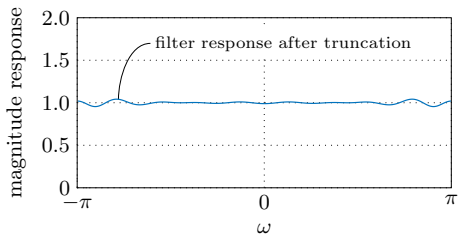
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Problem: Truncation Errors

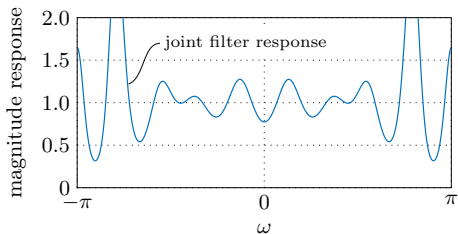


$$\mathbf{h}^{(1)} = \mathbf{h}^{(2)} = \dots = \mathbf{h}^{(M)}$$

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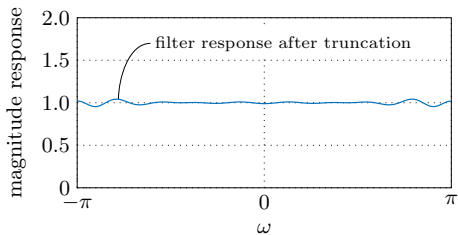


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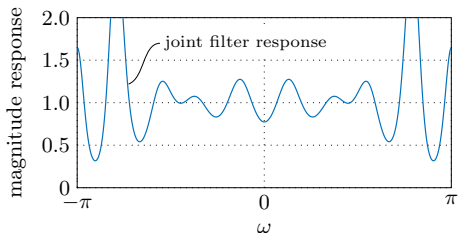


$$\mathbf{h}^{(1)} * \mathbf{h}^{(2)} * \dots * \mathbf{h}^{(M)}$$

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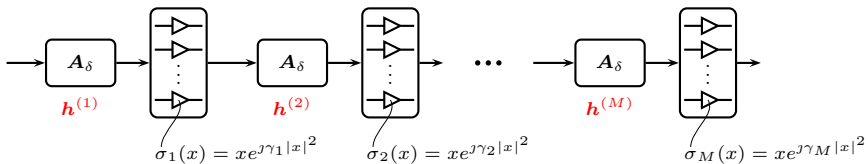
Our approach:
Optimize **all** M filters **jointly**

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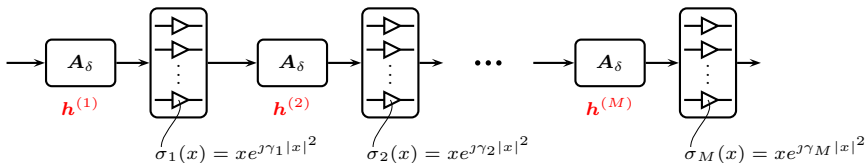
Joint Filter Optimization

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TensorFlow implementation of the computation graph $f_{\theta}(\mathbf{y})$:

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Deep learning of parameters $\theta = \{\mathbf{h}^{(1)}, \dots, \mathbf{h}^{(M)}\}$:

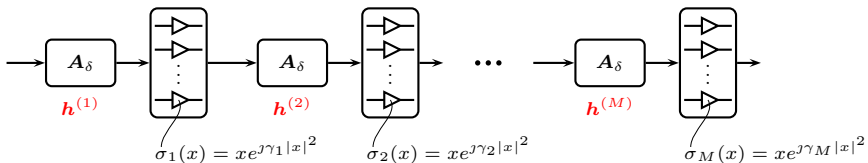
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mean squared error

using $\theta_{k+1} = \theta_k - \lambda \nabla_{\theta} g(\theta_k)$
Adam optimizer, fixed learning rate

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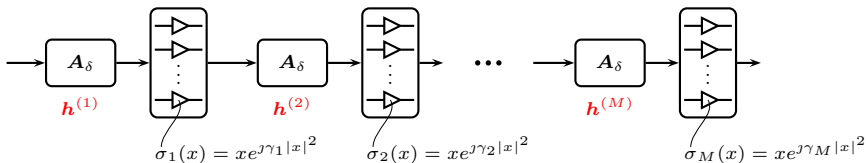
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Adam optimizer, fixed learning rate

- How to choose the starting point θ_0 and get short filters?

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mean squared error
Adam optimizer, fixed learning rate

- How to choose the starting point θ_0 and get short filters?
- Pre-optimization possible via multi-objective optimization problem

Iterative Filter Tap Pruning

$$\theta = \begin{cases} \mathbf{h}^{(1)} \\ \mathbf{h}^{(2)} \\ \vdots \\ \mathbf{h}^{(M)} \end{cases}$$

Iterative Filter Tap Pruning

← starting length $2K' + 1$ →

$$\theta = \left\{ \begin{array}{l} \mathbf{h}^{(1)} = (h_{K'}^{(1)} \cdots h_K^{(1)} \cdots h_1^{(1)} h_0^{(1)} h_1^{(1)} \cdots h_K^{(1)} \cdots h_{K'}^{(1)}) \quad \text{step 1} \\ \mathbf{h}^{(2)} = (h_{K'}^{(2)} \cdots h_K^{(2)} \cdots h_1^{(2)} h_0^{(2)} h_1^{(2)} \cdots h_K^{(2)} \cdots h_{K'}^{(2)}) \quad \text{step 2} \\ \vdots \\ \mathbf{h}^{(M)} = (h_{K'}^{(M)} \cdots h_K^{(M)} \cdots h_1^{(M)} h_0^{(M)} h_1^{(M)} \cdots h_K^{(M)} \cdots h_{K'}^{(M)}) \quad \text{step } M \end{array} \right.$$

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- Initially: constrained **least-squares coefficients** (LS-CO) [Sheikh et al., 2016]

Iterative Filter Tap Pruning

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 \leftarrow \text{target length } 2K + 1 \rightarrow \\
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 \mathbf{h}^{(2)} = (h_{K'}^{(2)} \cdots h_K^{(2)} \cdots h_1^{(2)} h_0^{(2)} h_1^{(2)} \cdots h_K^{(2)} \cdots h_{K'}^{(2)}) \quad \text{step 2} \\
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Iterative Filter Tap Pruning

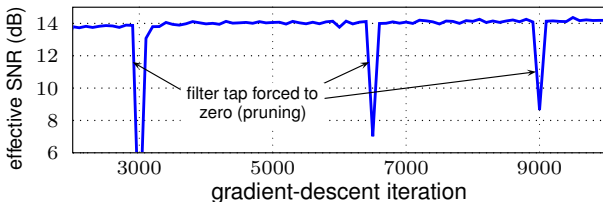
$$\begin{array}{c}
 \leftarrow \text{starting length } 2K' + 1 \rightarrow \\
 \leftarrow \text{target length } 2K + 1 \rightarrow \\
 \theta = \left\{ \begin{array}{l}
 \mathbf{h}^{(1)} = (\overset{\times}{h_{K'}^{(1)}} \cdots h_K^{(1)} \cdots h_1^{(1)} h_0^{(1)} h_1^{(1)} \cdots h_K^{(1)} \cdots \overset{\times}{h_{K'}^{(1)}}) \quad \text{step 1} \\
 \mathbf{h}^{(2)} = (\overset{\times}{h_{K'}^{(2)}} \cdots h_K^{(2)} \cdots h_1^{(2)} h_0^{(2)} h_1^{(2)} \cdots h_K^{(2)} \cdots \overset{\times}{h_{K'}^{(2)}}) \quad \text{step 2} \\
 \vdots \\
 \mathbf{h}^{(M)} = (h_{K'}^{(M)} \cdots h_K^{(M)} \cdots h_1^{(M)} h_0^{(M)} h_1^{(M)} \cdots h_K^{(M)} \cdots h_{K'}^{(M)}) \quad \text{step } M
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 \end{array}$$

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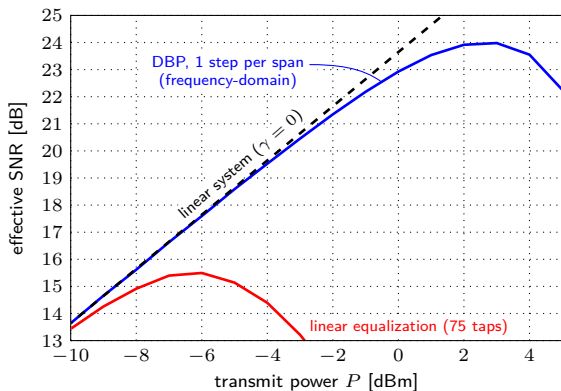
- Initially: constrained **least-squares coefficients** (LS-CO) [Sheikh et al., 2016]
- Typical **learning curve**:



Outline

1. Channel Modeling and the Nonlinear Schrödinger Equation
2. Connection to Deep Learning
3. Joint Filter Optimization and Pruning
- 4. Results**
5. Conclusions

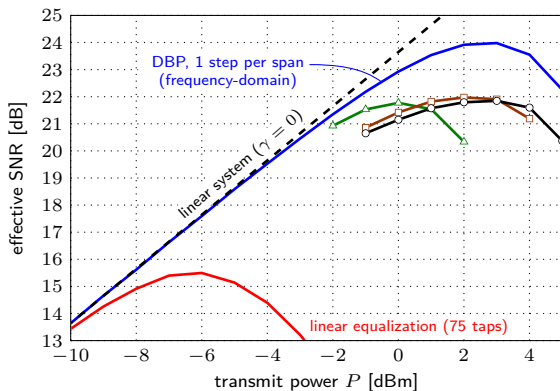
Optimization Results



Parameters similar to [Ip and Kahn, 2008]:

- 25×80 km SSFM
- Gaussian modulation
- RRC pulses (0.1 roll-off)
- 10.7 Gbaud
- 2 samples/symbol processing
- single channel, single pol.

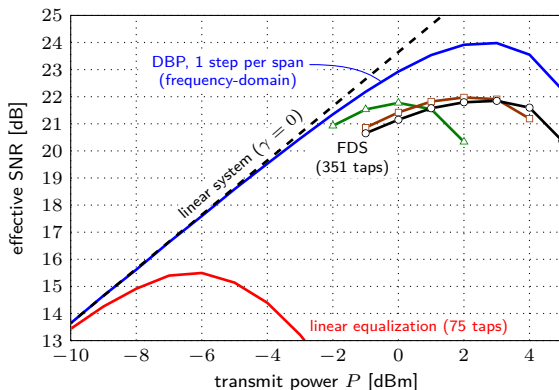
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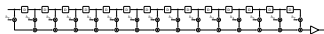
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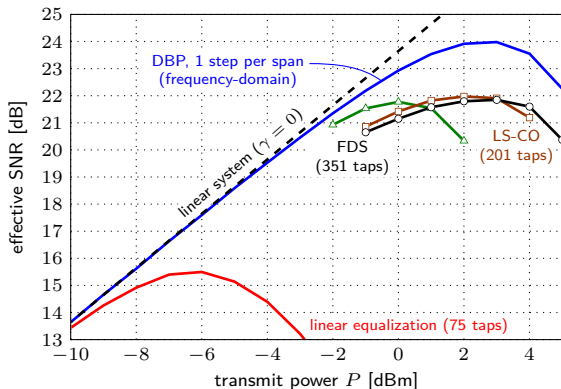
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FDS



... frequency-domain sampling (15 taps per step)

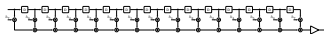
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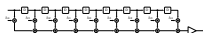
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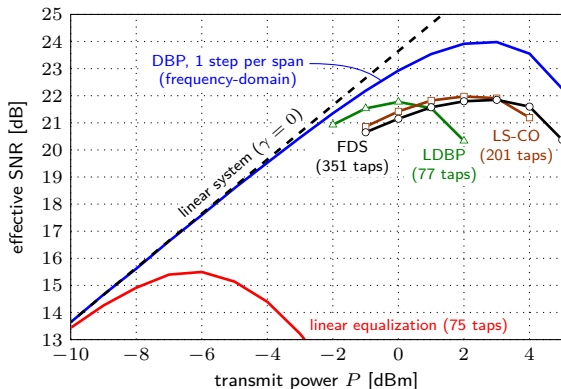
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LS-CO



... constrained least-squares [Sheikh et al., 2016] (9 taps per step)

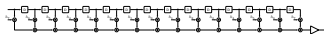
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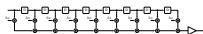
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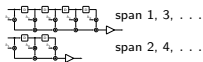
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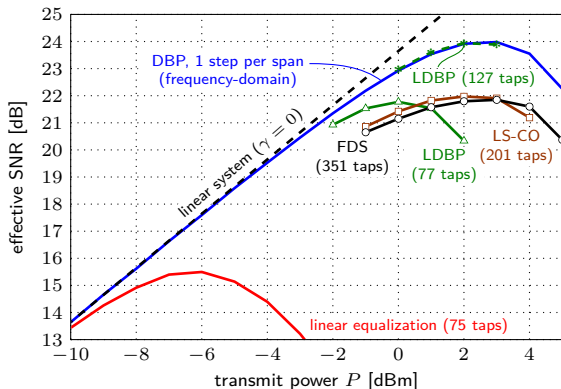
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LDBP



... deep learning and pruning (alternate 5 and 3 taps and use different coefficients in all steps)

Optimization Results



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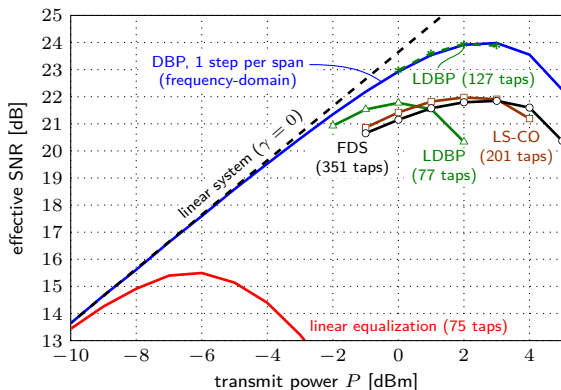
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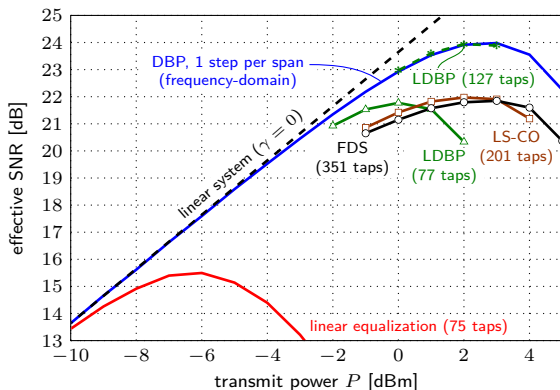
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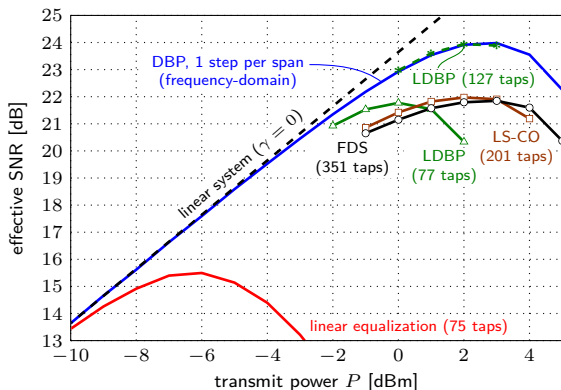


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- 28-nm CMOS synthesis results show ≈ 2 -fold power and area reduction compared to baseline filters (slightly different system parameters)
- Power consumption comparable to linear equalization in [Pillai et al., 2014],[Crivelli et al., 2014]
- Nonlinearities consume $< 20\%$ of the total power

Conclusions

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- We have addressed the problem of **inverting the nonlinear Schrödinger equation** for fiber-optic systems **in real time**
- Established numerical method (**split-step Fourier method**) leads to a computation graph reminiscent of a **deep feed-forward neural network**
- **Deep learning** in the resulting computation graph can be interpreted as a **joint filter (i.e., linear propagator) optimization problem**
- This approach requires **significantly fewer parameters than previous methods** \implies less complexity and power consumption

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Thank you!



References I



Crivelli, D. E., Hueda, M. R., Carrer, H. S., Del Barco, M., López, R. R., Gianni, P., Finochietto, J., Swenson, N., Voois, P., and Agazzi, O. E. (2014). Architecture of a single-chip 50 Gb/s DP-QPSK/BPSK transceiver with electronic dispersion compensation for coherent optical channels. *IEEE Trans. Circuits Syst. I: Reg. Papers*, 61(4):1012–1025.



Du, L. B. and Lowery, A. J. (2010). Improved single channel backpropagation for intra-channel fiber nonlinearity compensation in long-haul optical communication systems. *Opt. Express*, 18(16):17075–17088.



Essiambre, R.-J. and Winzer, P. J. (2005). Fibre nonlinearities in electronically pre-distorted transmission. In *Proc. European Conf. Optical Communication (ECOC)*, Glasgow, UK.



Fougstedt, C., Mazur, M., Svensson, L., Eliasson, H., Karlsson, M., and Larsson-Edefors, P. (2017a). Time-domain digital back propagation: Algorithm and finite-precision implementation aspects. In *Proc. Optical Fiber Communication Conf. (OFC)*, Los Angeles, CA.



Fougstedt, C., Svensson, L., Mazur, M., Karlsson, M., and Larsson-Edefors, P. (2017b). Finite-precision optimization of time-domain digital back propagation by inter-symbol interference minimization. In *Proc. European Conf. Optical Communication*, Gothenburg, Sweden.



Goldfarb, G. and Li, G. (2009). Efficient backward-propagation using wavelet-based filtering for fiber backward-propagation. *Opt. Express*, 17(11):814–816.

References II



Häger, C. and Pfister, H. D. (2018).
Nonlinear interference mitigation via deep neural networks.
In Proc. Optical Fiber Communication Conf. (OFC), San Diego, CA.



Ip, E. and Kahn, J. M. (2008).
Compensation of dispersion and nonlinear impairments using digital backpropagation.
J. Lightw. Technol., 26:3416–3425.



Li, L., Tao, Z., Dou, L., Yan, W., Oda, S., Tanimura, T., Hoshida, T., and Rasmussen, J. C. (2011).
Implementation efficient nonlinear equalizer based on correlated digital backpropagation.
In Proc. Optical Fiber Communication Conf. (OFC), page OWW3, Los Angeles, CA.



Li, X., Chen, X., Goldfarb, G., Mateo, E., Kim, I., Yaman, F., and Li, G. (2008).
Electronic post-compensation of WDM transmission impairments using coherent detection and digital signal processing.
Opt. Express, 16(2):880–888.



Martins, C. S., Bertignono, L., Nespola, A., Carena, A., Guiomar, F. P., and Pinto, A. N. (2018).
Efficient time-domain DBP using random step-size and multi-band quantization.
In Proc. Optical Fiber Communication Conf. (OFC), San Diego, CA.





Napoli, A., Maalej, Z., Sleiffer, V. A. J. M., Kuschnerov, M., Rafique, D., Timmers, E., Spinnler, B., Rahman, T., Coelho, L. D., and Hanik, N. (2014).
Reduced complexity digital back-propagation methods for optical communication systems.
J. Lightw. Technol., 32(7).



Paré, C., Villeneuve, A., Bélanger, P.-A. A., and Doran, N. J. (1996).
Compensating for dispersion and the nonlinear Kerr effect without phase conjugation.
Optics Letters, 21(7):459–461.

References III

-  Pillai, B. S. G., Sedighi, B., Guan, K., Anthapadmanabhan, N. P., Shieh, W., Hinton, K. J., and Tucker, R. S. (2014). End-to-end energy modeling and analysis of long-haul coherent transmission systems. *J. Lightw. Technol.*, 32(18):3093–3111.
-  Rafique, D., Zhao, J., and Ellis, A. D. (2011). Digital back-propagation for spectrally efficient wdm 112 gbit/s pm m-ary qam transmission. *Opt. Express*, 19(6):5219–5224.
-  Roberts, K., Li, C., Strawczynski, L., O'Sullivan, M., and Hardcastle, I. (2006). Electronic precompensation of optical nonlinearity. *IEEE Photon. Technol. Lett.*, 18(2):403–405.
-  Secondini, M., Rommel, S., Meloni, G., Fresi, F., Forestieri, E., and Poti, L. (2016). Single-step digital backpropagation for nonlinearity mitigation. *Photon. Netw. Commun.*, 31(3):493–502.
-  Sheikh, A., Fougstedt, C., Graell i Amat, A., Johannisson, P., Larsson-Edefors, P., and Karlsson, M. (2016). Dispersion compensation FIR filter with improved robustness to coefficient quantization errors. *J. Lightw. Technol.*, 34(22):5110–5117.
-  Yan, W., Tao, Z., Dou, L., Li, L., Oda, S., Tanimura, T., Hoshida, T., and Rasmussen, J. C. (2011). Low complexity digital perturbation back-propagation. In *Proc. European Conf. Optical Communication (ECOC)*, page Tu.3.A.2, Geneva, Switzerland.
-  Zhu, L., Li, X., Mateo, E., and Li, G. (2009). Complementary FIR filter pair for distributed impairment compensation of WDM fiber transmission. *IEEE Photon. Technol. Lett.*, 21(5):292–294.