On Parameter Optimization for Staircase Codes

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Outline

Staircase Codes (and Product Codes)

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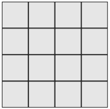
- Start with a binary linear code $\mathcal{C}(n, k, d_{\text{min}})$ as a "building block"

Staircase Codes (and Product Codes)

rectangular array [Elias, 1954]



Example: $n = 4$

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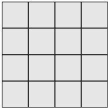

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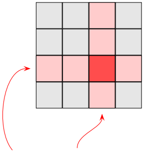

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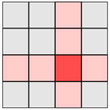

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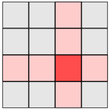

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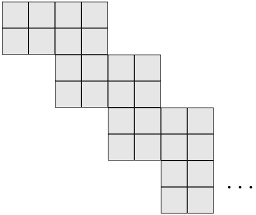

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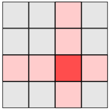

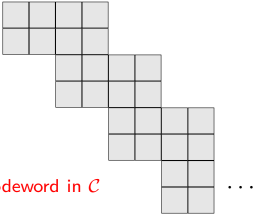

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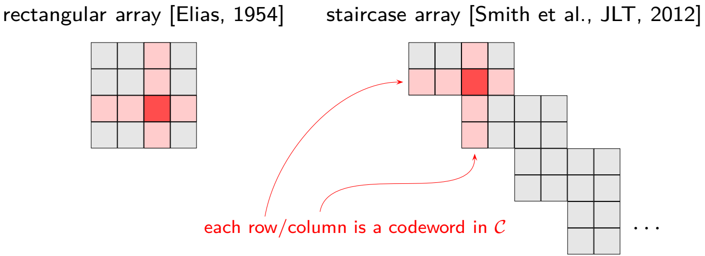

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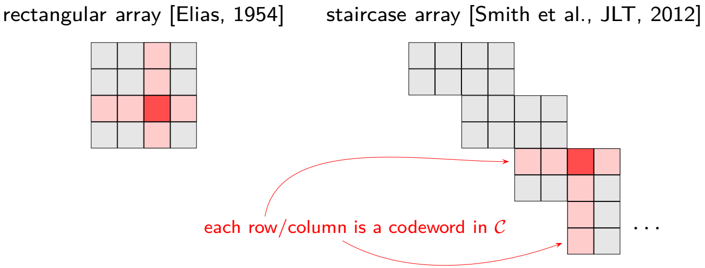

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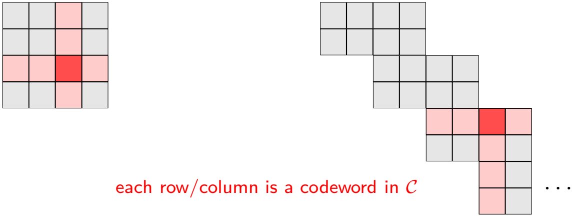

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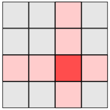

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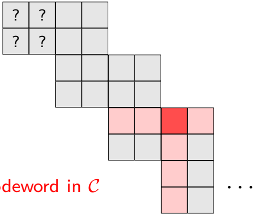

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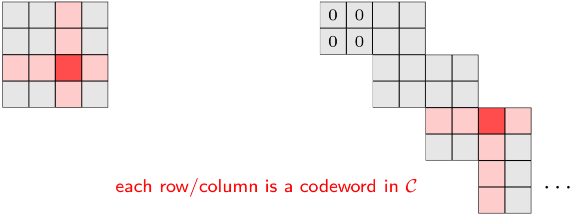

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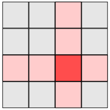

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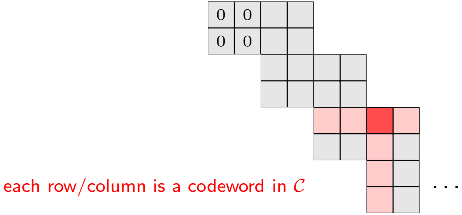

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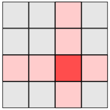

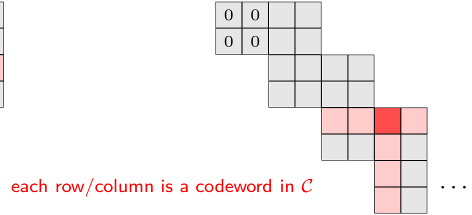

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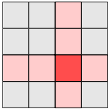

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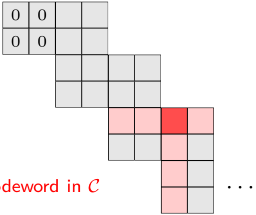

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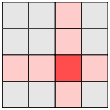

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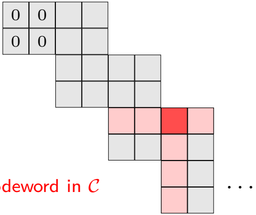

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Problem Formulation

For fixed OH, find a "good" triple (ν, t, s).

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- Connect staircase codes to spatially-coupled generalized LDPC (SC-GLDPC) code ensemble in [Jian et al., ISIT, 2012]
- Use density evolution and ensemble thresholds to optimize parameters, can account for miscorrections assuming extrinsic message passing (EMP) [Jian et al., ISIT, 2012]

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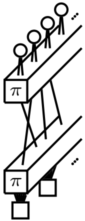

Tanner graph

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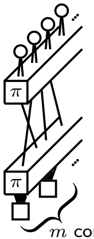

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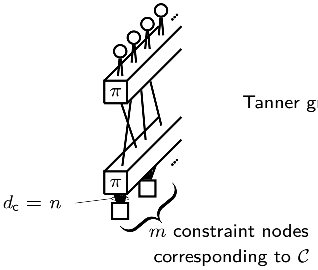

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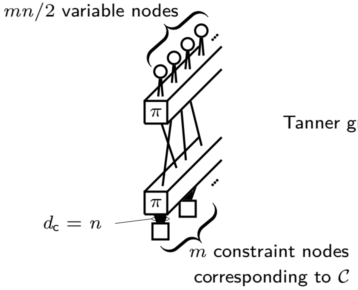

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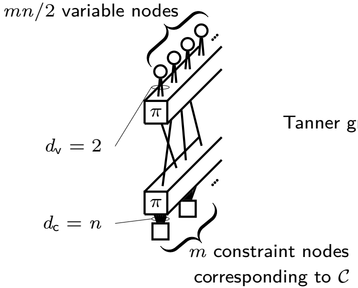

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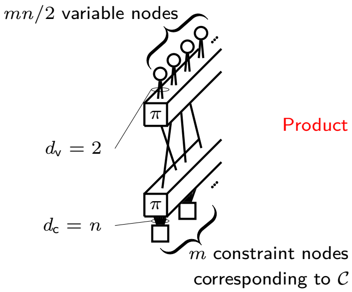

Product code: $m = 2n$, "structured" permutations π

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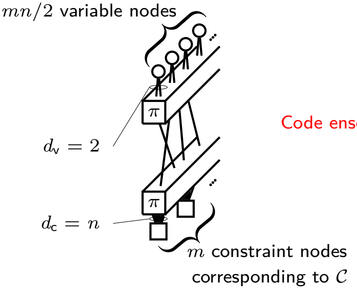

Code ensemble: set of codes defined by all possible π

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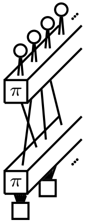

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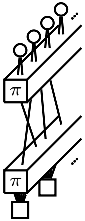

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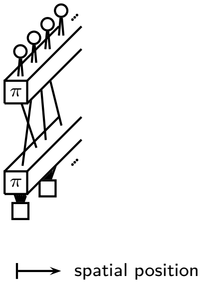

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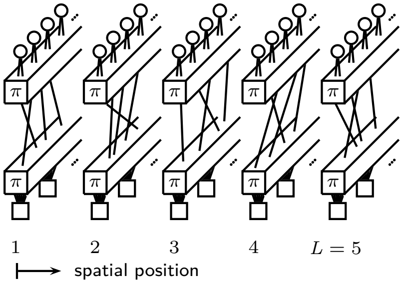

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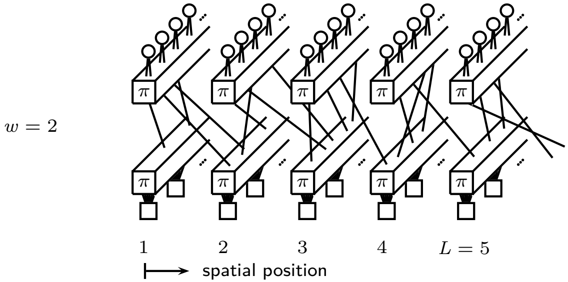

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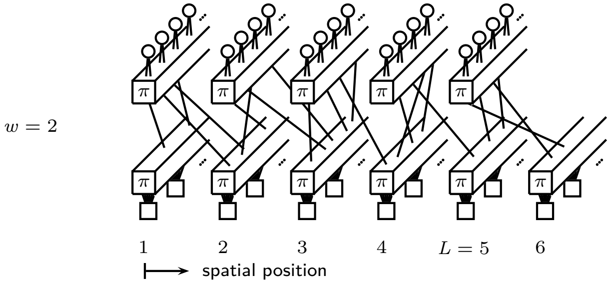

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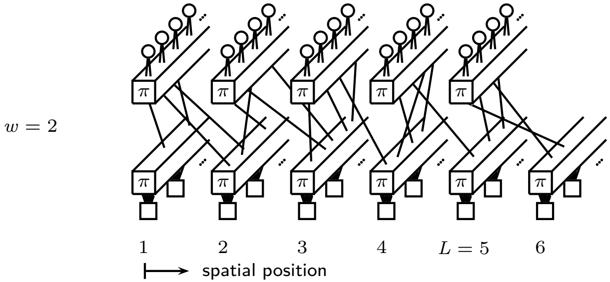

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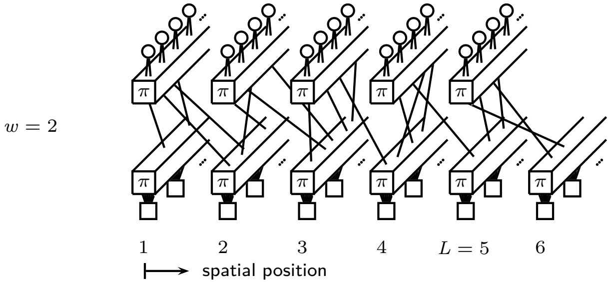

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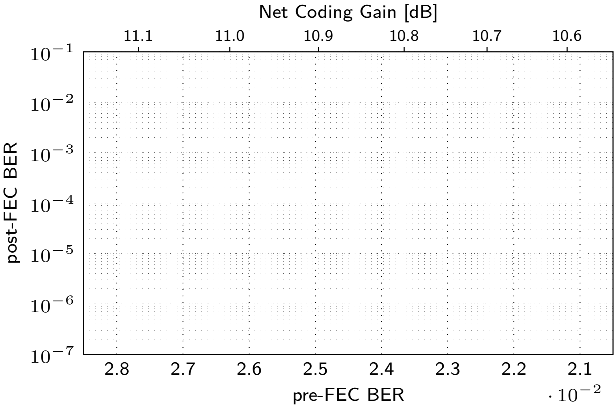
- Asymptotic $(m \to \infty)$ ensemble behavior can be analyzed via density evolution (DE) assuming extrinsic message passing (EMP)

Example (OH $= 33.33\%$): Density Evolution and Thresholds



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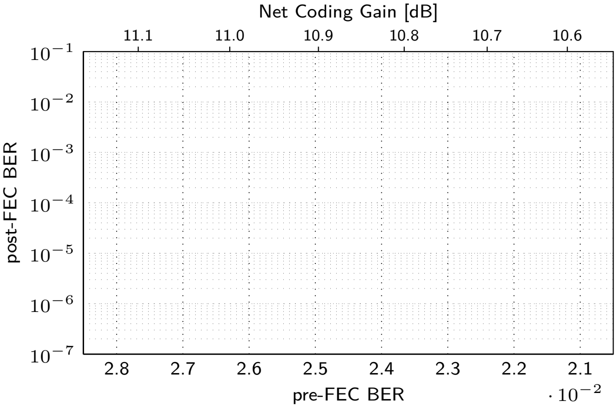

- $\mathcal{C}_1$ with $(\nu, t, s) = (9, 5, 151)$ *[Zhang and Kschischang, JLT, 2014]

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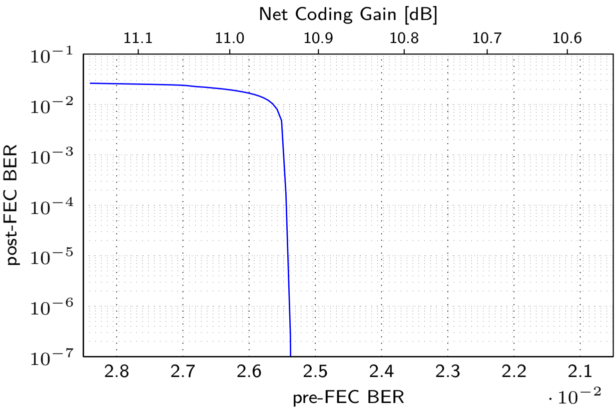

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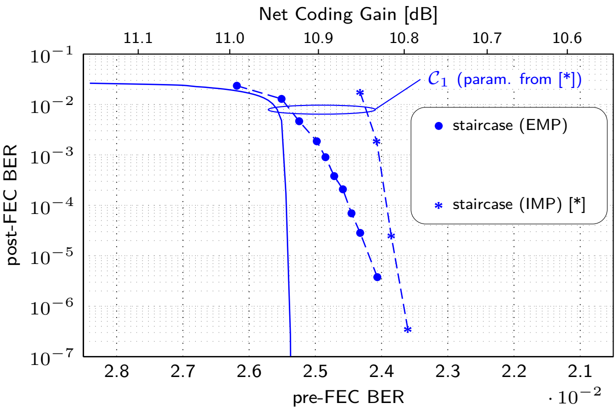

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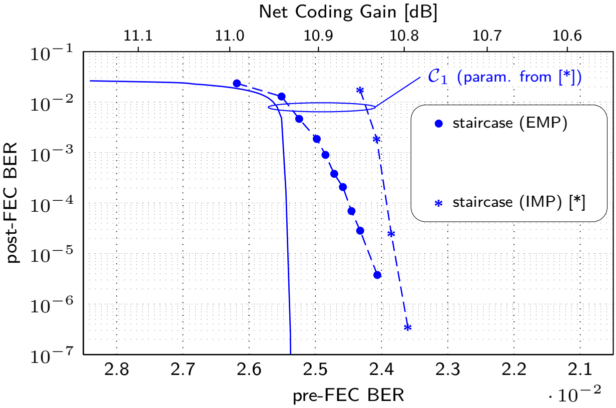

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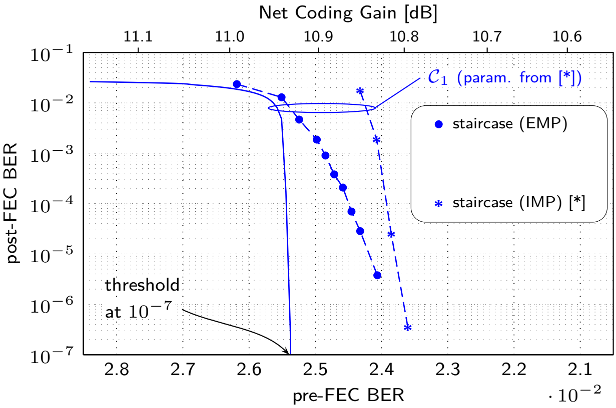

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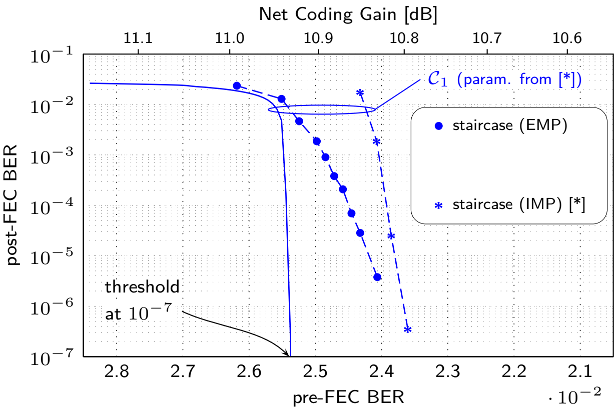

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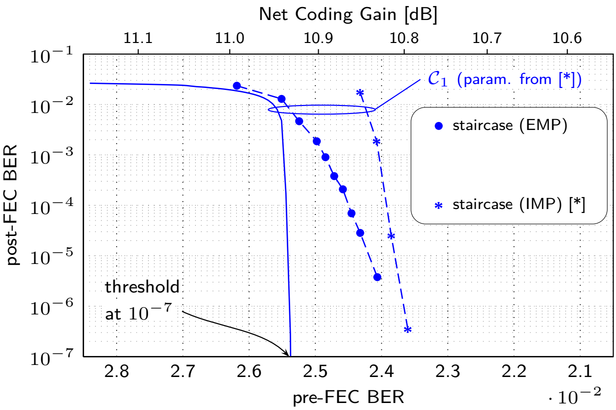

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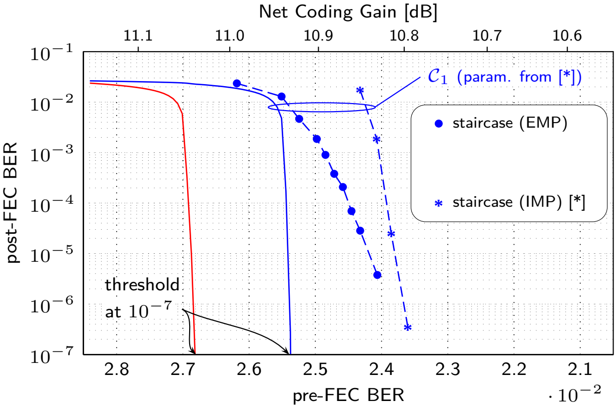

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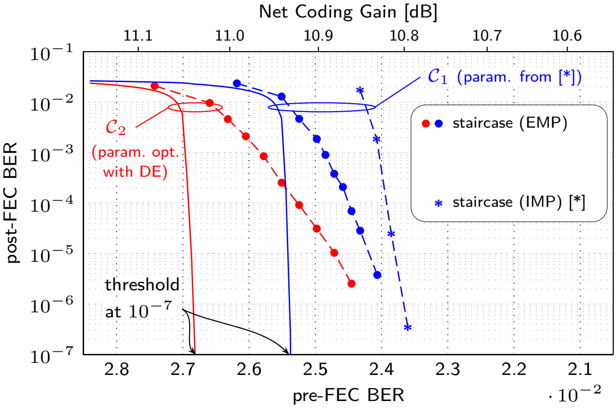

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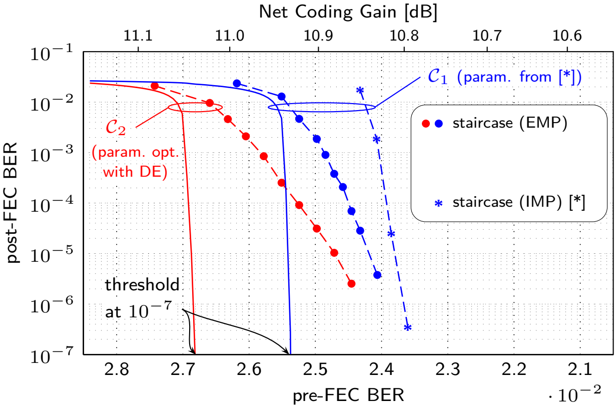

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- Staircase codes with $\mathcal{C}_1$ and $\mathcal{C}_2$ have different slopes ⇒ DE gain prediction not preserved

Staircase Array with Multiple Row/Column Constraints

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- “steepness” of BER curve determined by m, but fixed in the original construction

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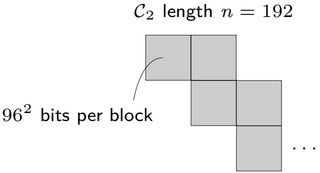

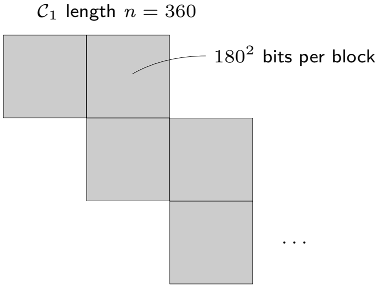

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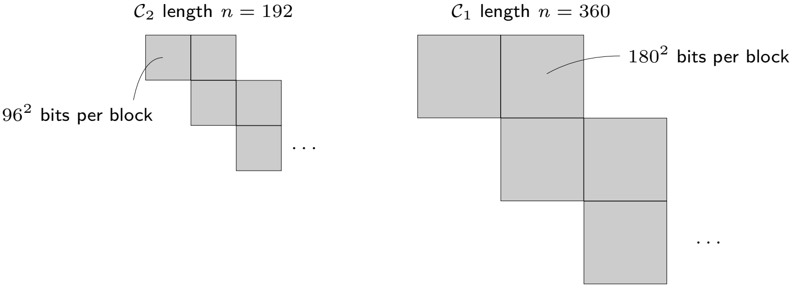

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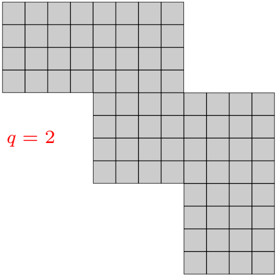

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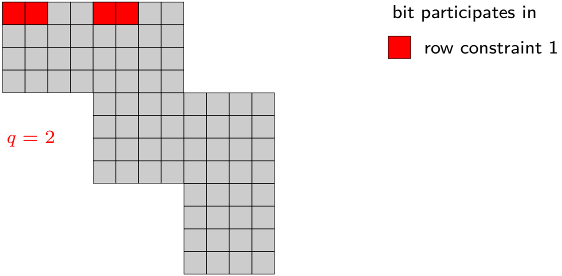

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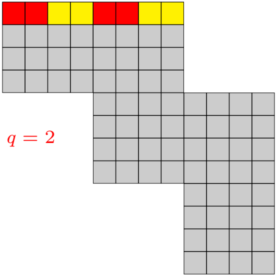

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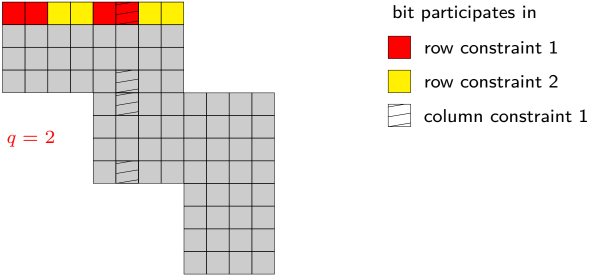

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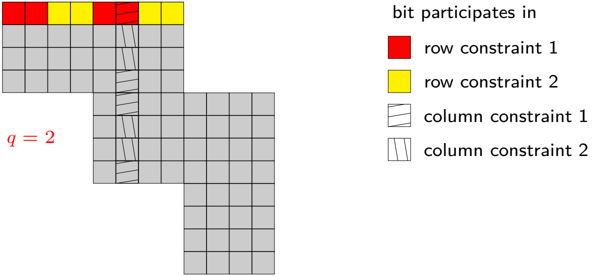

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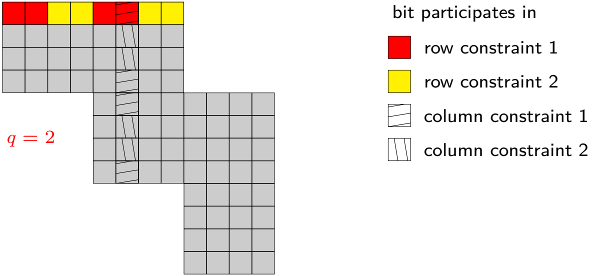

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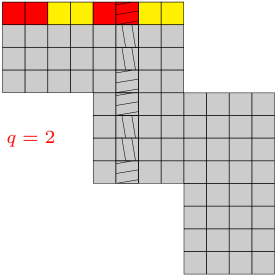

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- column constraint 1
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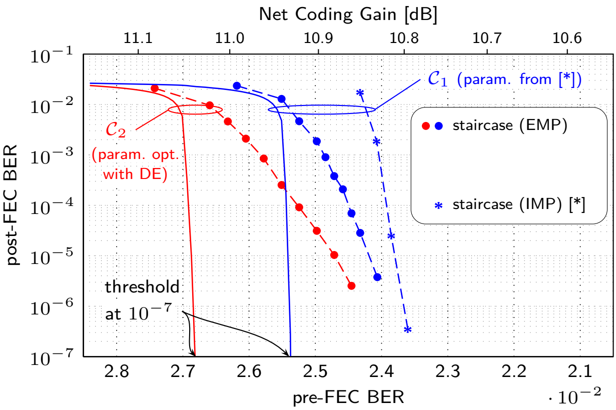
- The type of lifting preserves the staircase array structure and time-invariant encoding/decoding operations

Example (OH $= 33.33\%$): Extended Code Construction



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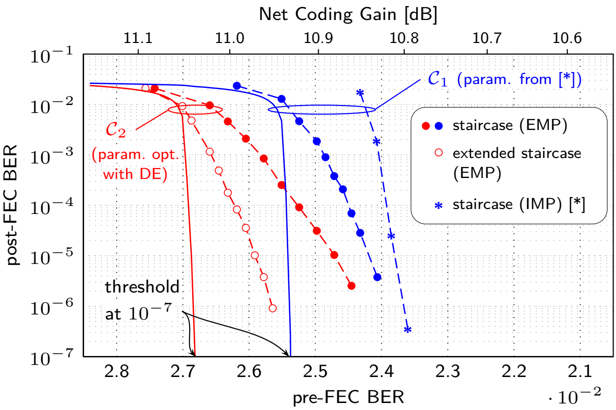

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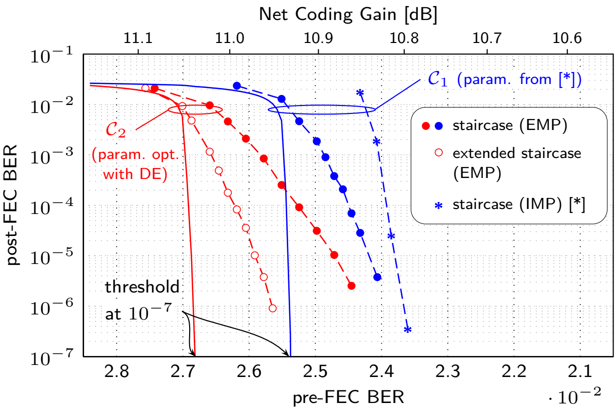

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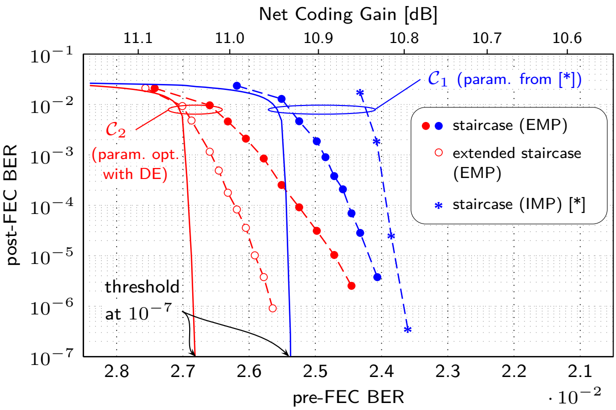

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- Steeper waterfall performance at the expense of a larger staircase block size $2 \cdot n/2 = 192$
- Staircase code with $\mathcal{C}_1$ has block size $n/2 = 180$

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Thank you!

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