Spatially-Coupled Codes for Optical Communications: State-of-the-Art and Open Problems

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Motivation

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Coding scheme	algebraic codes RS $(255, 239)$ hard	concatenated codes RS+BCH, RS+RS hard	iteratively decodable codes block turbo codes & LDPC codes soft
NCG (10^{-13})	~ 5.8 dB	$7 - 9$ dB	~ 10 dB

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  - Drawbacks: error floor, non-universal.

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A new coding paradigm: Spatially-coupled LDPC codes

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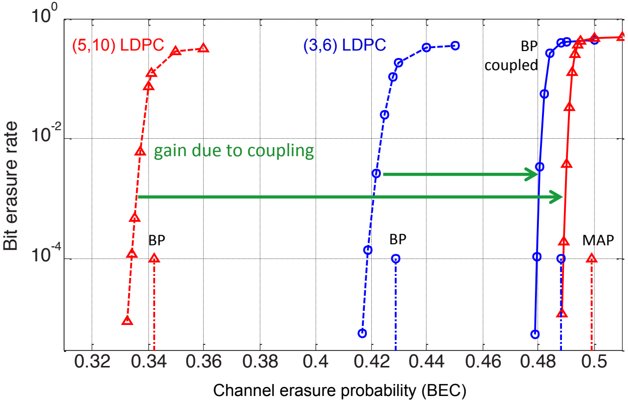
Main principle

The BP threshold saturates to the optimal MAP threshold of the underlying LDPC block code ensemble.

Spatial coupling gain



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In this talk

Spatially-coupled codes: promising candidates for future fiber-optical systems

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Outline:

1. Basics of SC-LDPC Codes
2. SC-LDPC Codes and high-order modulation (SDD).
3. Spatially-coupled codes for HDD (staircase codes and extended staircase codes)

Spatially-coupled LDPC codes

Spatially-coupled LDPC codes

- A SC-LDPC code is constructed from an (regular) LDPC code applying a copy & coupling procedure.

Spatial coupling: Code construction

$M = 10$ code bits

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

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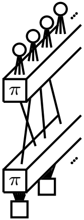
M code bits



parity-checks

(Tanner graph)

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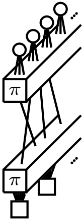

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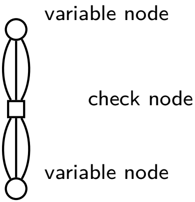

protograph rate-$1/2$
$(3, 6)$-regular LDPC code

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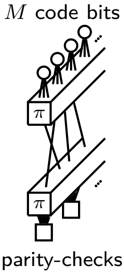

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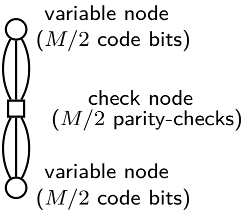

protograph rate-$1/2$ $(3, 6)$-regular LDPC code (length M bits)

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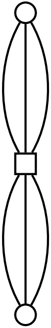
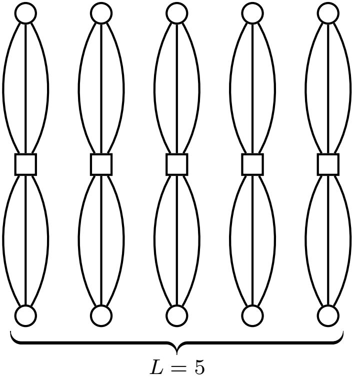


protograph rate-$1/2$ $(3,6)$-regular LDPC code

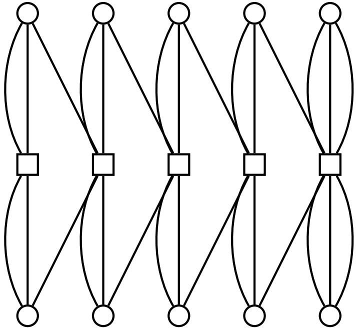
Spatial coupling: Code construction

copy the protograph L times



Spatial coupling: Code construction

connect (couple) the protographs



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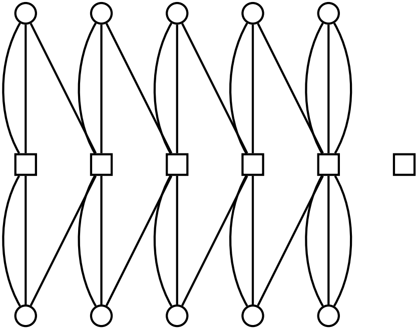

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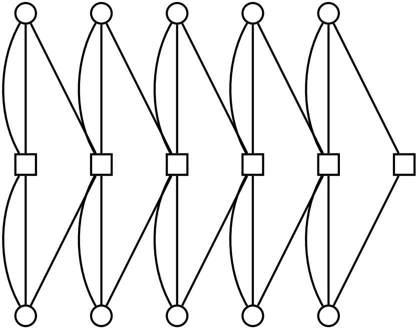

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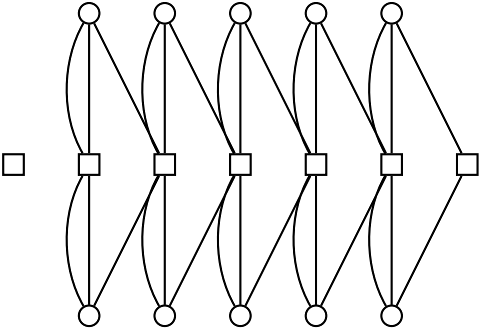

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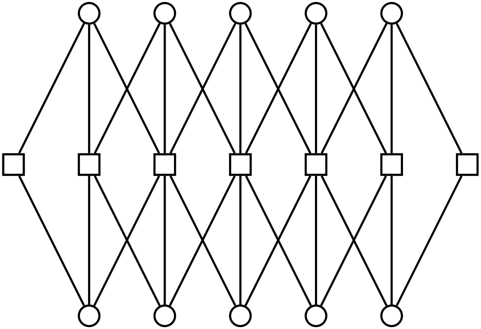

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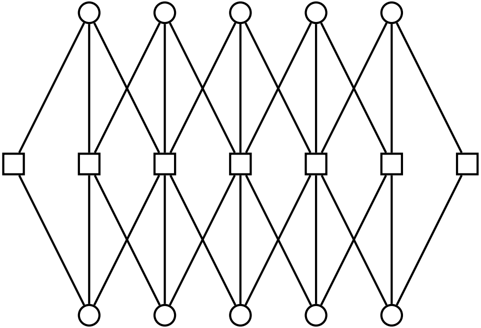
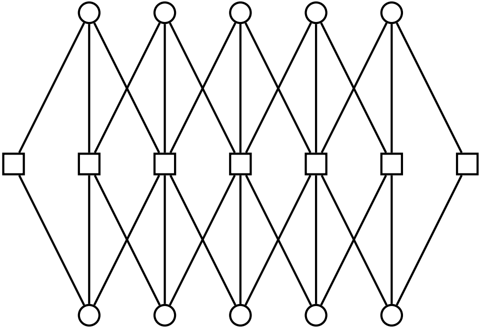


(terminated) coupled chain of $L = 5$ LDPC codes

Spatial coupling: Code construction



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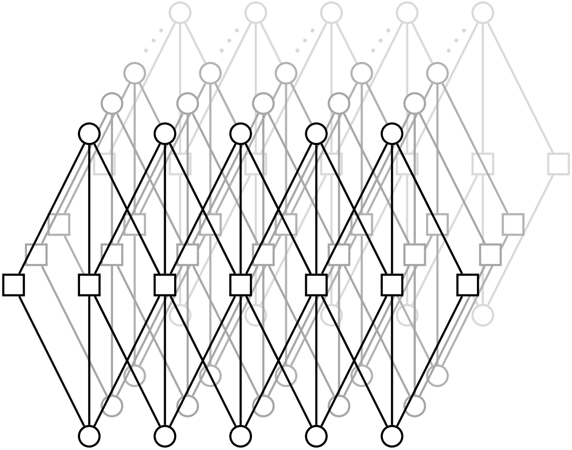
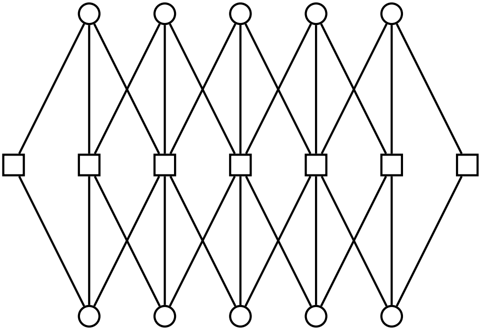


Tanner graph $(3, 6, L = 5)$ terminated SC-LDPC code

Spatial coupling: Code construction



regular graph...except at the boundaries

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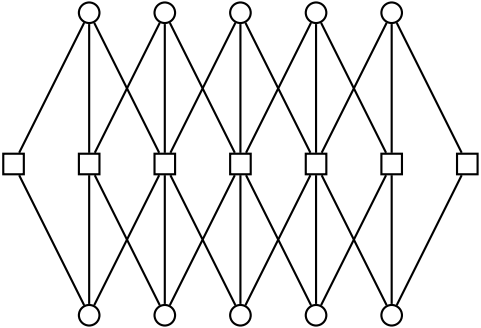

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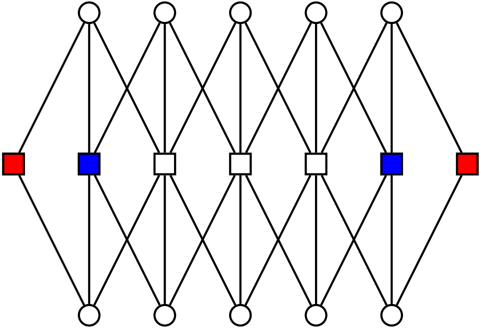

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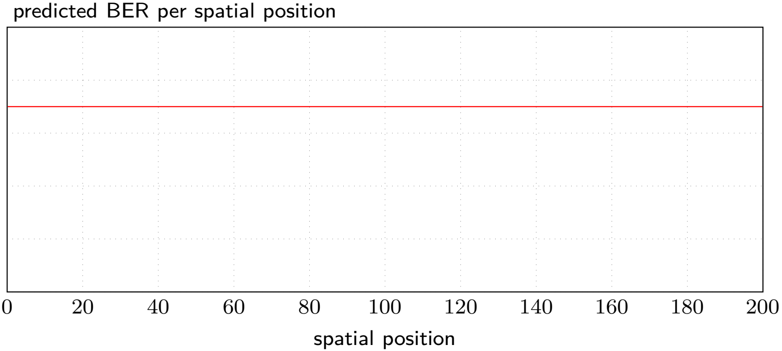
Decoding Wave (terminated SC-LDPC code)



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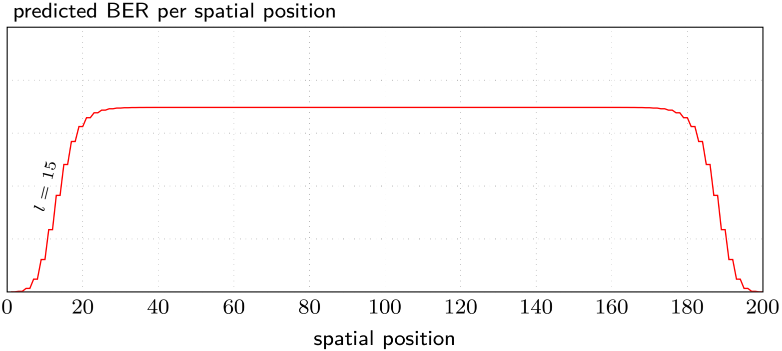

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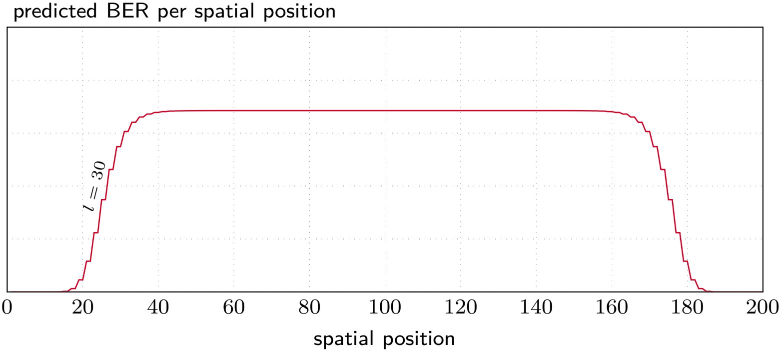

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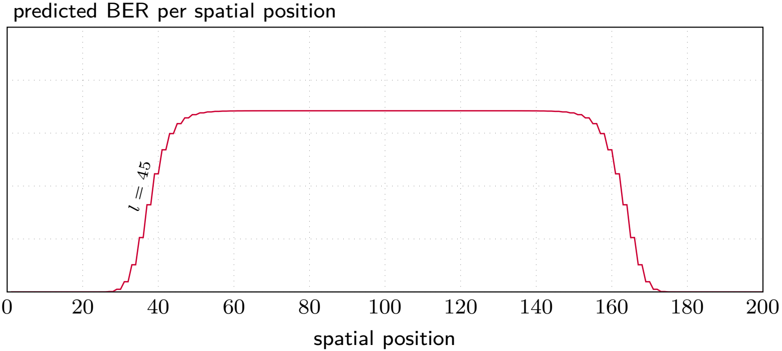

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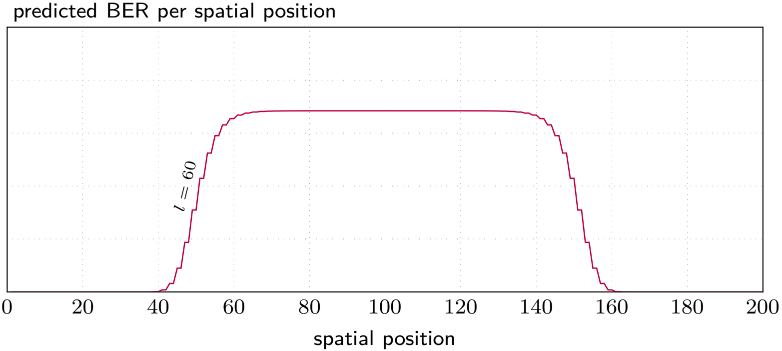

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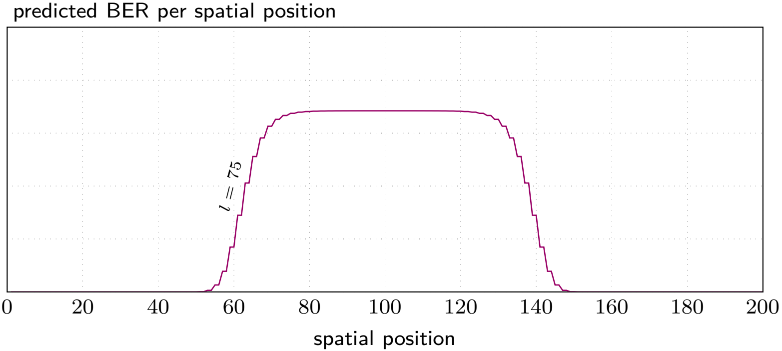

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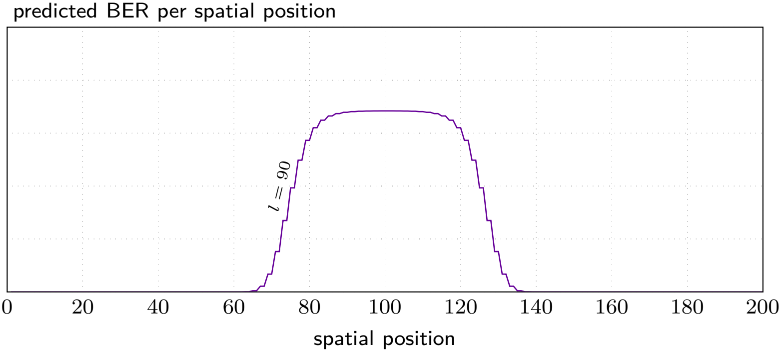

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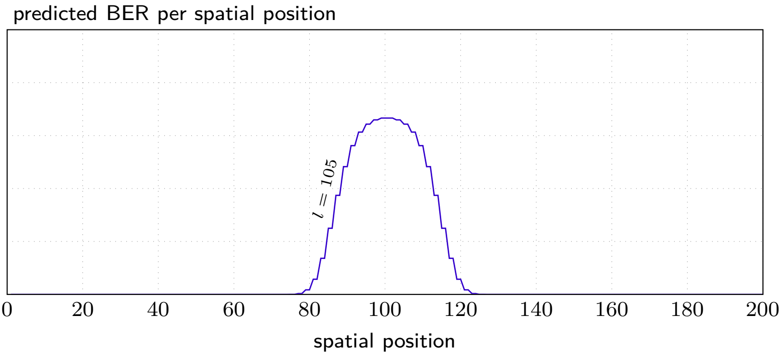

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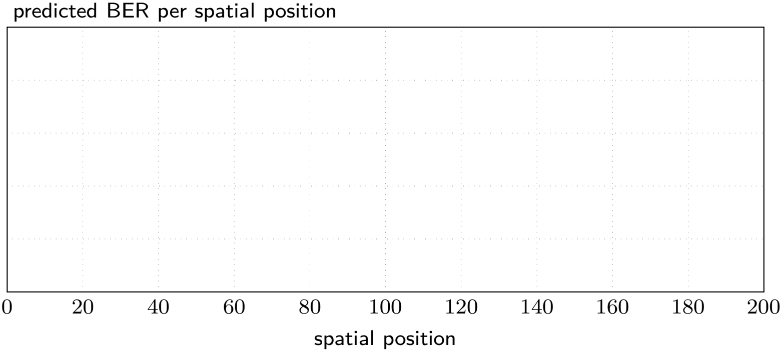
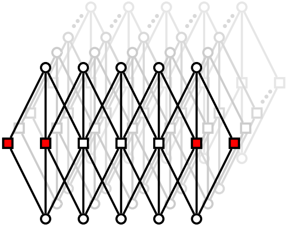


Successful decoding!

Example: Terminated



check node degrees — slightly irregular

performance — capacity-approaching (wave effect)

linear distance growth

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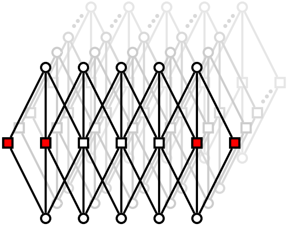
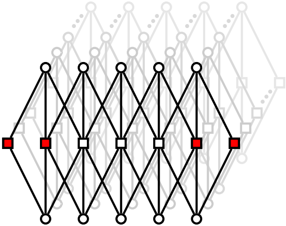


check node degrees	slightly irregular
performance	capacity-approaching (wave effect)
	linear distance growth
rate	$R(L) = R - R_{\text{loss}}(L)$ (larger OH)

Example: Terminated Tailbiting



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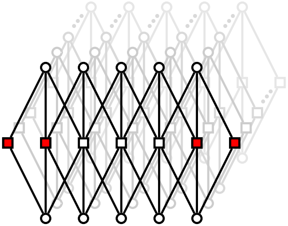

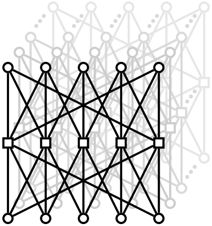

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	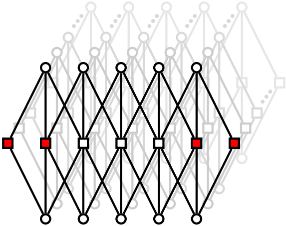	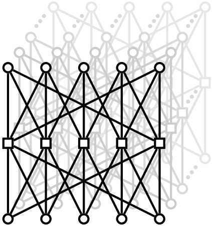
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	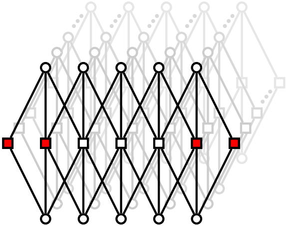	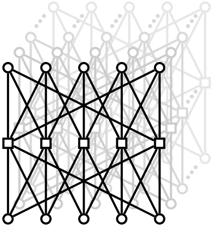
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	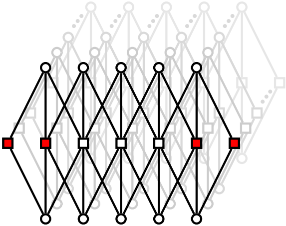	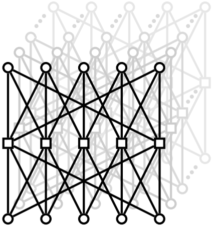
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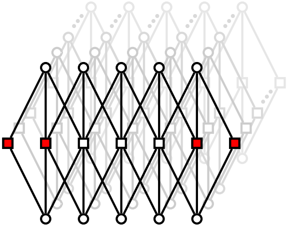
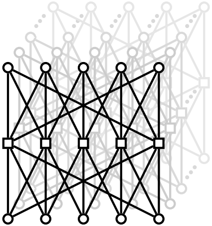
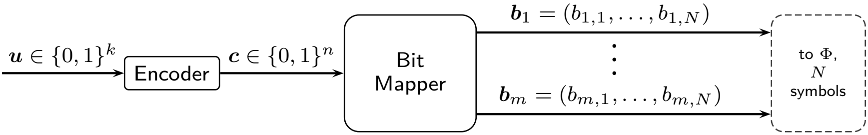
		
check node degrees	slightly irregular	regular
performance	capacity-approaching (wave effect)	comparable to regular LDPC (no wave effect)
	linear distance growth	linear distance growth
rate	$R(L) = R - R_{\text{loss}}(L)$ (larger OH)	R (no rate loss)

Coded Molulation: Bit Mapper



- Bit mapper determines allocation of the coded bits to the modulation bits.

[1] C. Häger, A. Graell i Amat, F. Brännström, A. Alvarado, E. Agrell, "Terminated and Tailbiting Spatially-Coupled Codes with Optimized Bit Mappings for Spectrally Efficient Fiber-Optical Systems," *IEEE/OSA J. Lightwave Technology, April 2015.*

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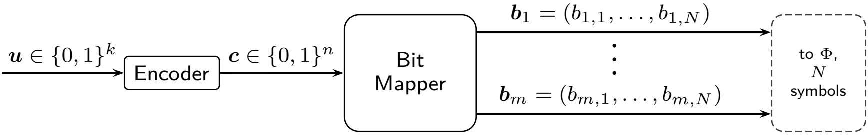

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- Baseline bit mapper: sequential or random.

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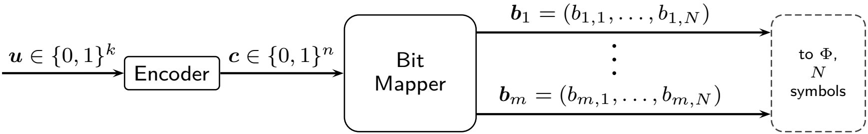

- Bit mapper determines allocation of the coded bits to the modulation bits.
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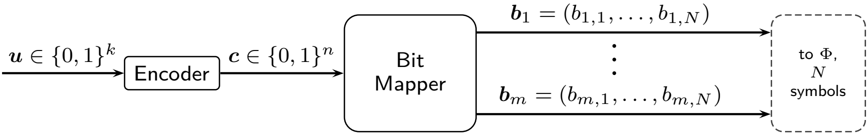

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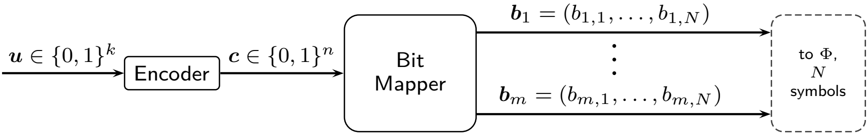

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- Bit mapper is optimized to optimize the decoding threshold.

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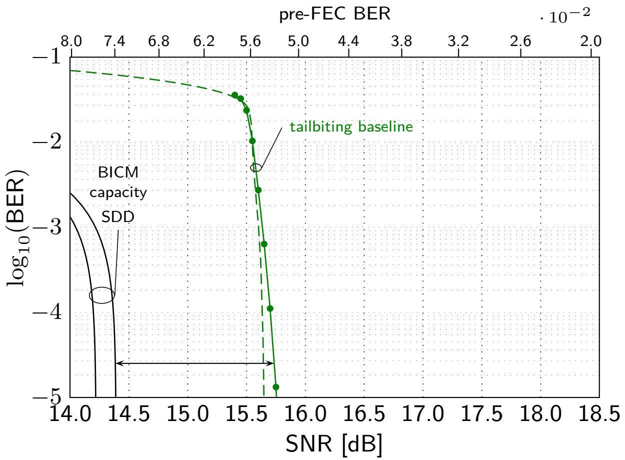

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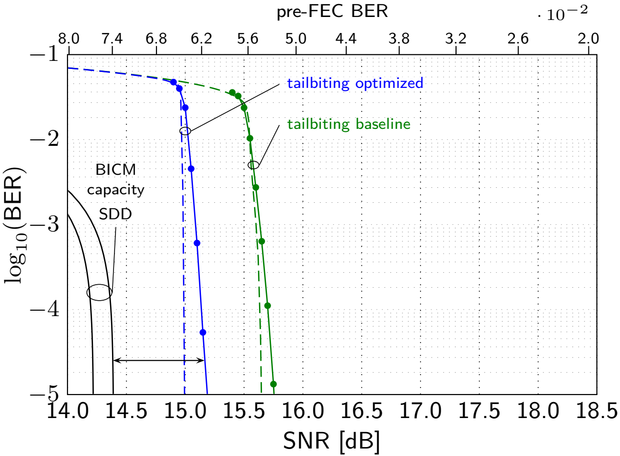

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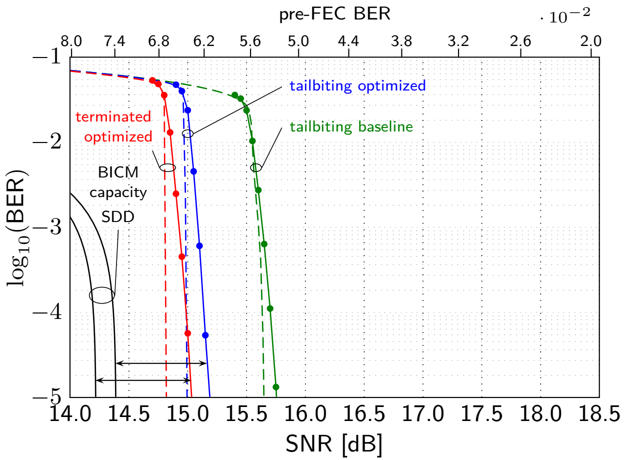

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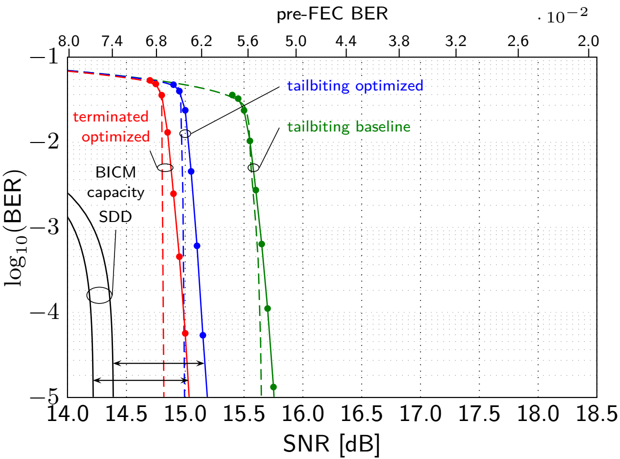

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- Approximately the same gap to capacity for both optimized systems.

Spatial coupling for HDD

Spatial coupling is a very general concept!

- Spatially-coupled codes for HDD (e.g., staircase codes).

Staircase Codes (and Product Codes)

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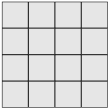
- Start with a binary linear code $\mathcal{C}(n, k, d_{\text{min}})$ as a "building block"

Staircase Codes (and Product Codes)

rectangular array [Elias 1954]



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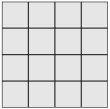

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each row/column is a codeword in $\mathcal{C}$
($2n$ code constraints in total)

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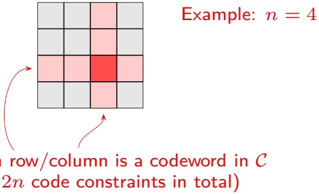

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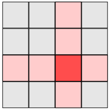

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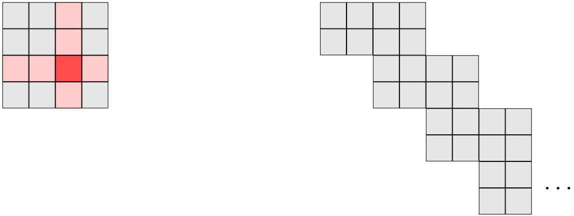

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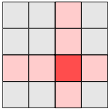

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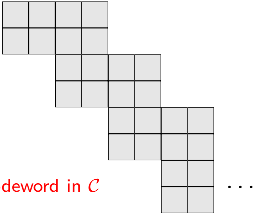

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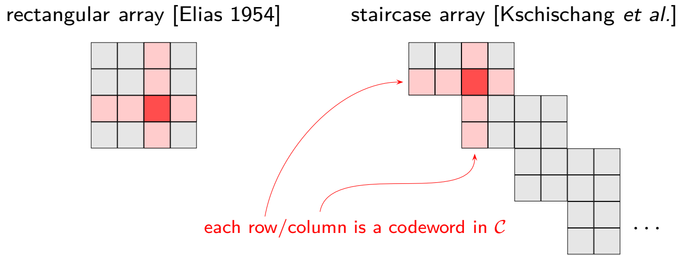

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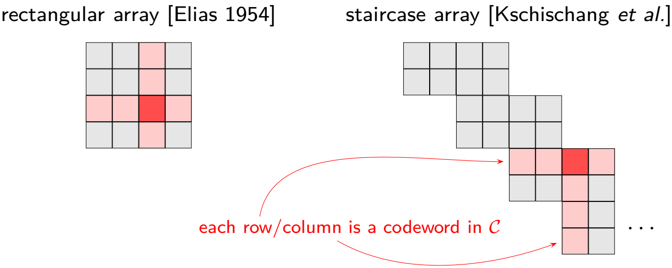

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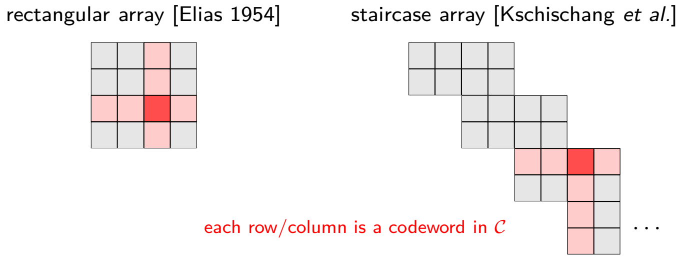

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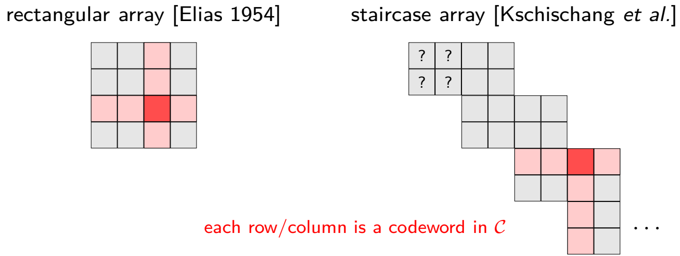

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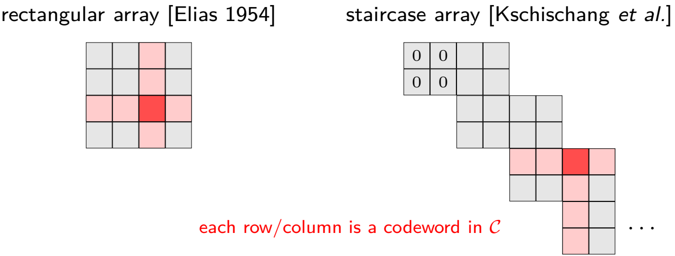

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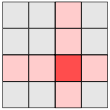

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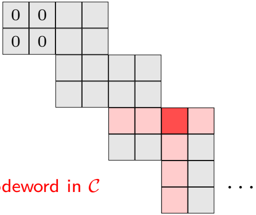

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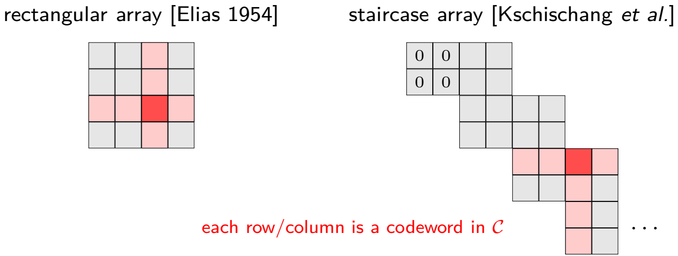

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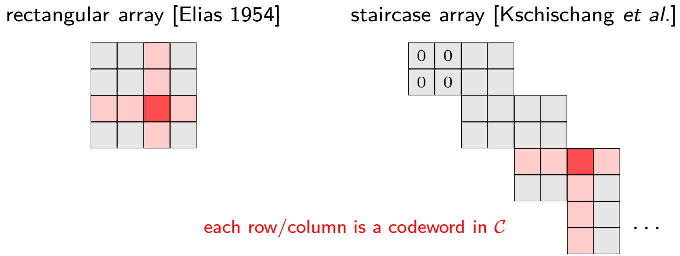

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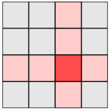

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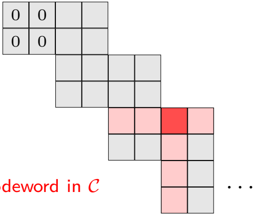

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Problem Formulation

For fixed OH, find a "good" triple (ν, t, s).

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Staircase codes as SC-GLDPC codes

Observation

Staircase codes can be seen as a class of spatially-coupled generalized LDPC (SC-GLDPC) codes!

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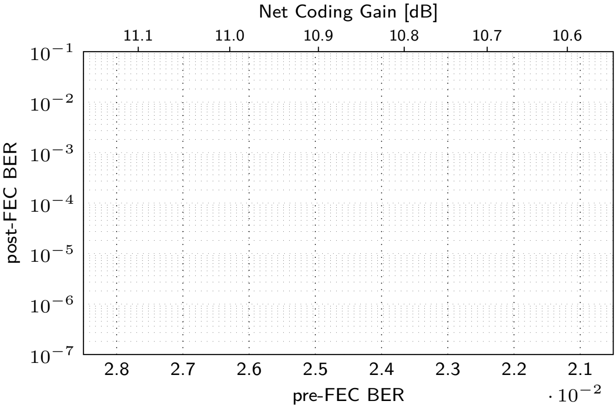

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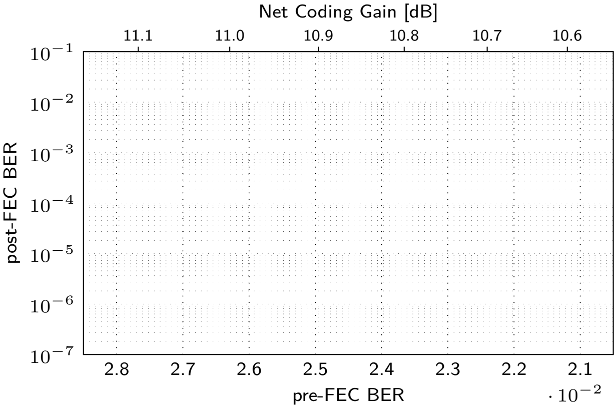

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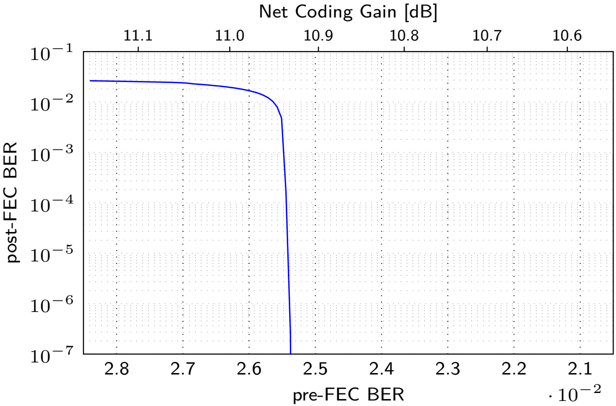

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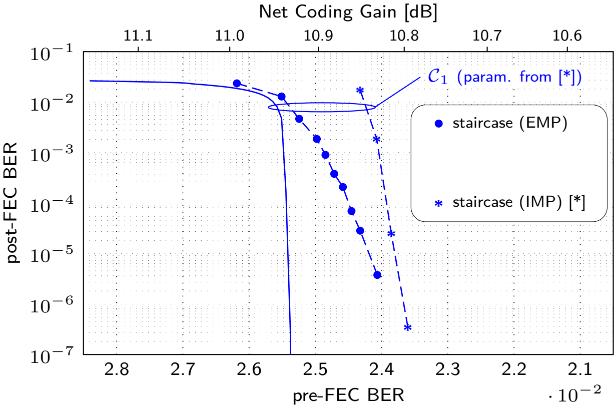

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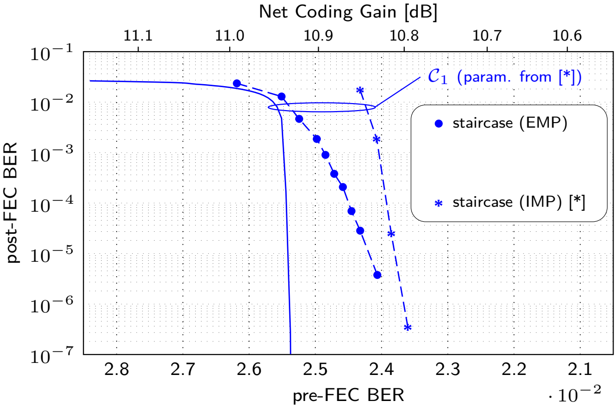

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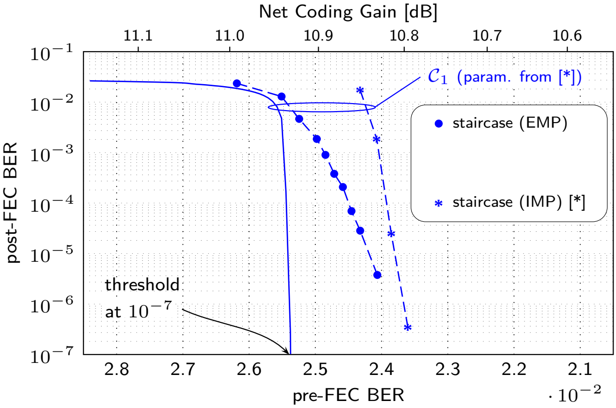

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- Use decoding thresholds for parameter optimization

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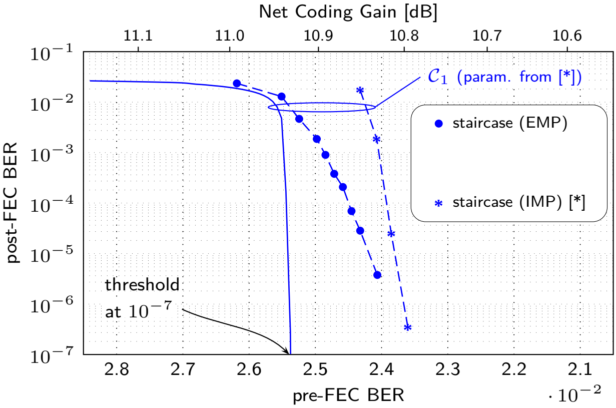

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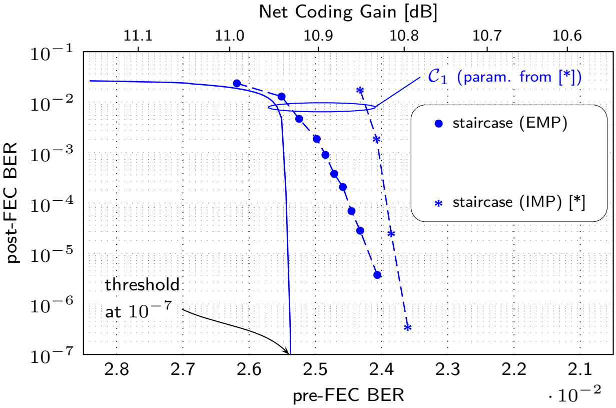

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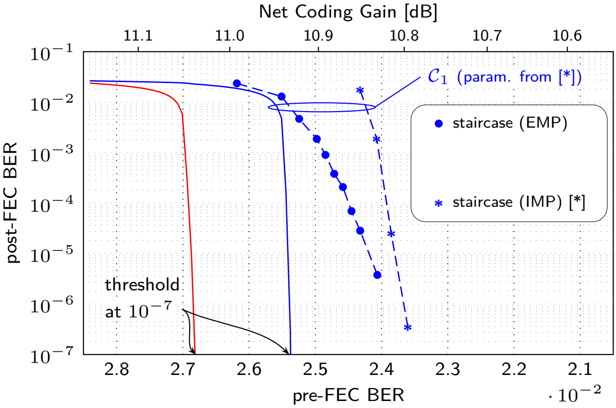

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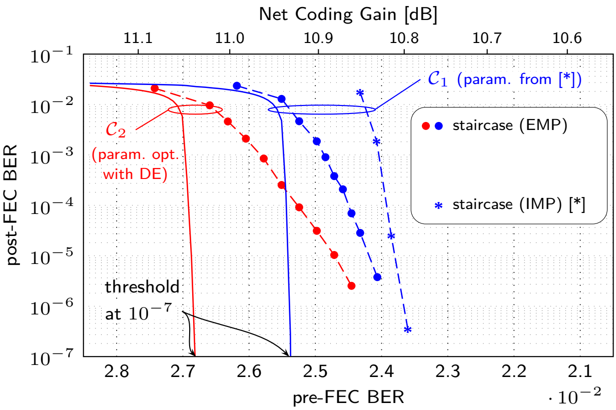

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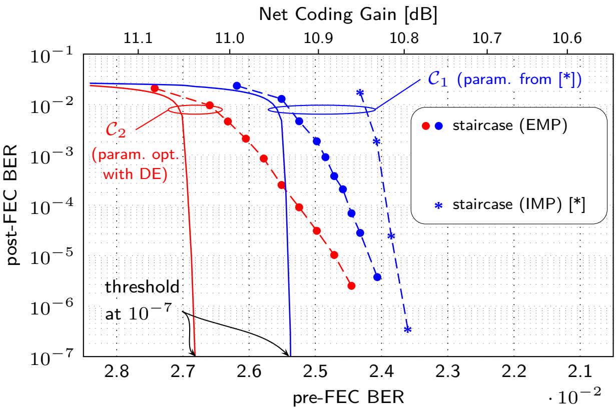

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- Staircase codes with $\mathcal{C}_1$ and $\mathcal{C}_2$ have different slopes $\Rightarrow$ DE gain prediction not preserved

Staircase Array with Multiple Row/Column Constraints

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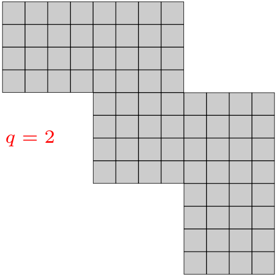

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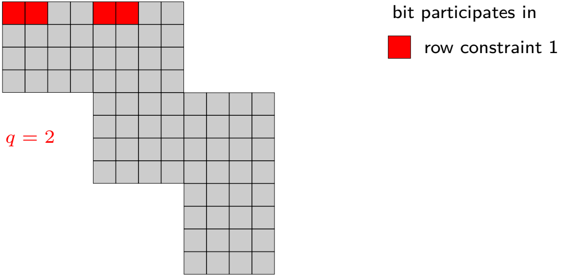

- Allow for $q > 1$ code constraints in each row/column of the staircase array $\longrightarrow$ improves steepness of BER curve.

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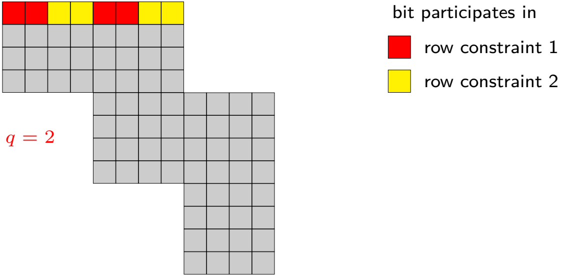

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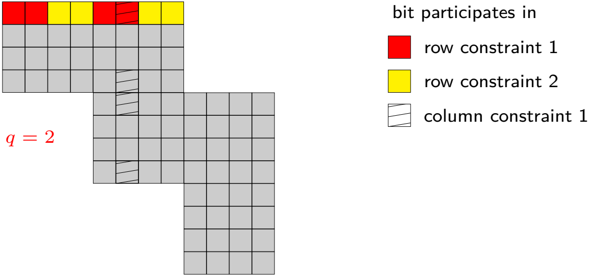

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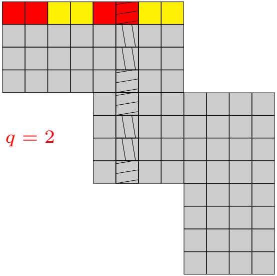

bit participates in

- row constraint 1
- row constraint 2
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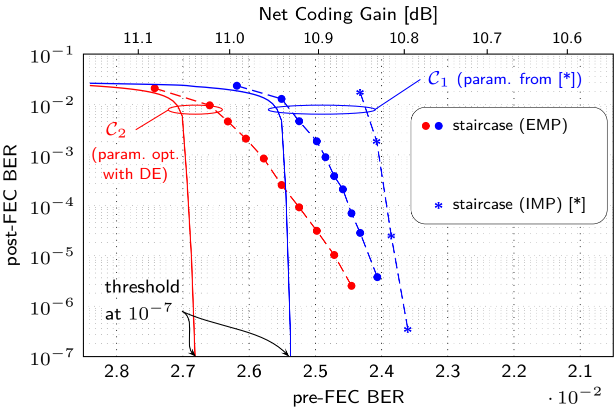
Example (OH $= 33.33\%$): Extended Code Construction



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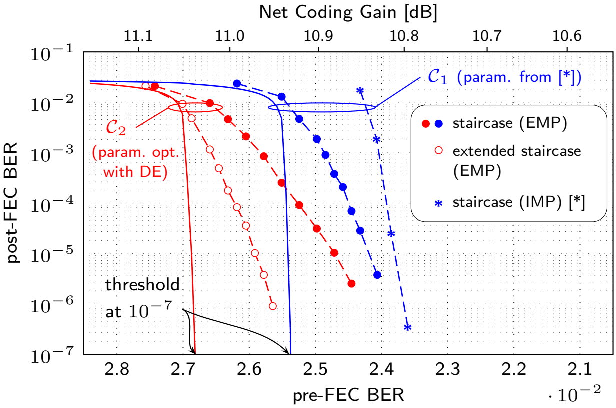

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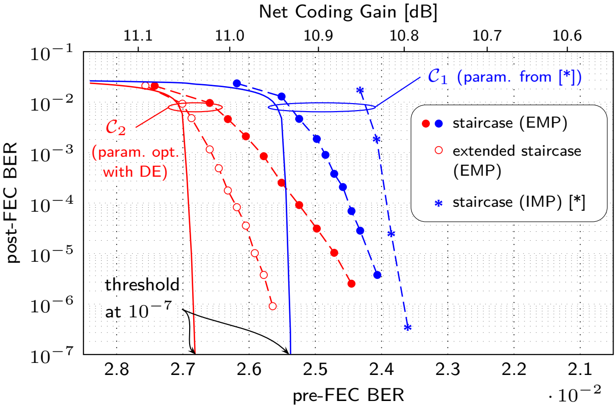

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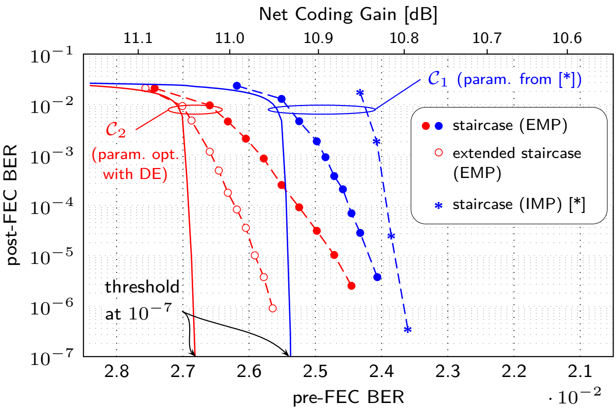

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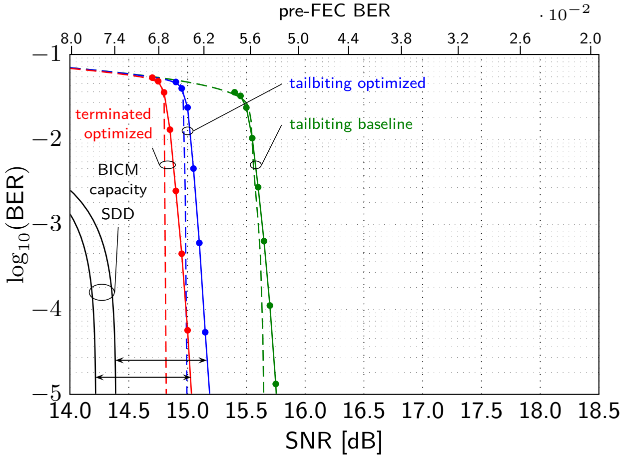
Performance for soft and hard decision decoding



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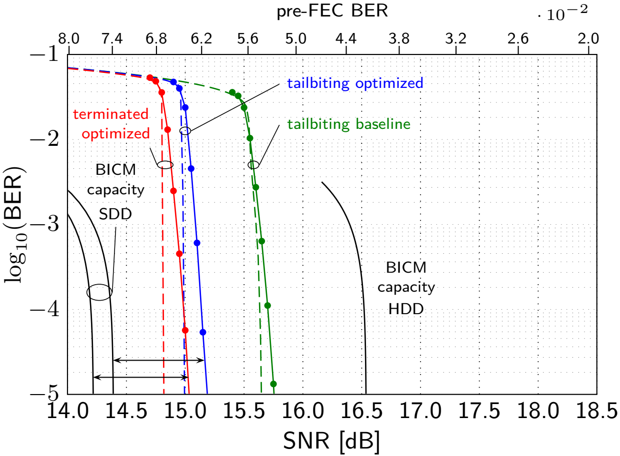

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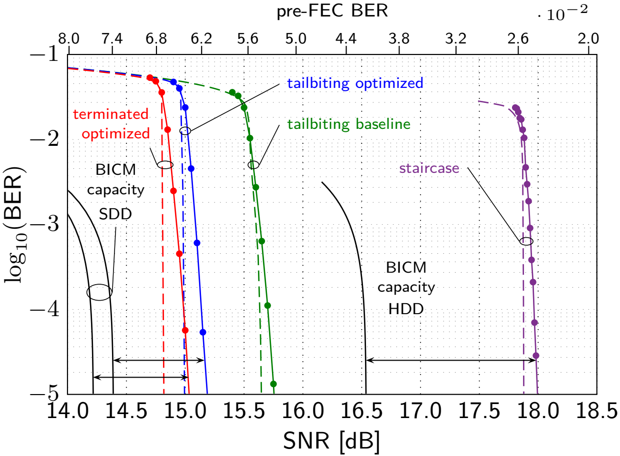

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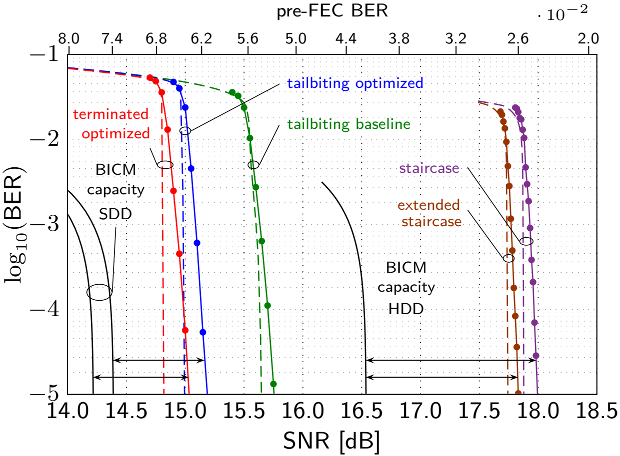

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Thank you!

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